Mathematical models for urban growth

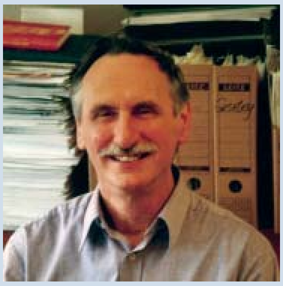
Is there a way to describe the growth and expansion of a city? To answer this question, Sergio Albeverio, professor at the Academy of Architecture of the Università della Svizzera italiana (USI), has devised a mathematical model where the complex processes of an urban area's growth and evolution have been converted into algorithms. Under the name of ACME (Cellular automata and Master Equations), this project is subsidised by the Swiss National Science Foundation (SNF) and by USI.

The evolutionary process of a city is a fairly unpredictable mechanism: it is far from linear and often defeats any planning attempts by specialists. The growth of a city is determined by several parameters from all kinds of contexts - cultural, historical, economic, and political - and planning strategies plug into these. We are all familiar with some of the factors that forge urban space: communication routes condition the location and development of industrial parks; these in turn lead to people's migrations and to the positioning of new homes for workmen. Hence, new buildings are put up, others are vacated, whole districts originally designed for residential purposes end up as office or commercial space. These transformations are influenced by other factors, too, such as the market value of rents and land, as well as mortgage rates. Over time, one can see entire city areas emerge and change shape, with the consequent shift and flux of people and activities. The illusion that one can control and direct a city's system by applying a small number of rational principles has on many occasions turned out to be quite disastrous. *"Like dreams"* - wrote Italo Calvino - "*cities are bred out of fears and desires*"; and like dreams urban development, too, may elude the control of those who wish to plan its contents and evolution. From this point of view, the city becomes a complex system capable of organising itself, of expanding and adapting according to its own set of rules.

Different ways of capitalising on the land, and how they interact.

So simple, yet so complex!

The algorithms developed by ACME are nothing more than long mathematical formulae, but the results are quite remarkable. *"One of the most striking aspects* - declares Sergio Albeverio (in the picture) - *is precisely the lack of proportion between the simplicity of the evolutionary rules prevailing at the microscopic level and the complexity of behaviours at the global level"*. In one sense the city takes on a new meaning, it becomes a dynamic entity, while resting on very simple regulations. *"Compare it with what goes on in our brain: no neurone by itself has the incredible power and faculties that the brain releases. The latter spring from the complex design of the system as a whole. The way in which the parts interact with the whole generates qualities that each part alone does not have".*



ACME: A city in an algorithm

At USI's Academy of Architecture, Professor Sergio Albeverio's research team faced a challenge: to design a mathematical model to look into urban development, and to work out how a city responds and adjusts to possible planning initiatives. The ACME project has several possible applications: planners might use the mathematical model to predict growth and to anticipate medium-term development paths. Also, modelling might be applied to pre-empt possible repercussions of certain infrastructural work: new communication routes, alterations to the town-planning scheme, etc. Another field of application consists in assessing the effectiveness of the instruments used in regional planning. By means of algorithms one can make projections, hence observe whether planning strategies reach the expected goals.

Urban space evolves like a living organism

A city grows, changes and achieves organisation under the influence of those same social, economic, and cultural dynamics that mould the structure of the region. A city may be compared to a living organism, which interacts with all the elements of the world surrounding it. A number of factors play a role in its evolutionary processes on all scales, from the individual's human scale to the global scale. Indeed a house is built by an individual owner, who nonetheless is expected to abide by the rules of the town-planning scheme as well as by the socio-economic situation of the place. For example, the owner will decide to build either offices or residential flats depending on the opportunities available and on the context where the building will stand. Other factors, such as the existence of communication routes, shops, services, or green spaces will have an impact on the evolution of a certain plot of land. The mathematical model devised by the Academy of Architecture's researchers draws inspiration from cellular automata, an analytical method used to study the emergence of complex structures on the basis of a small number of rules at the microscopic level. As is the case with cellular automata, in cities, too, there is no central authority to oversee the development of the whole system according to a pre-conceived plan. The global characteristics of the system are the result of interactions taking place at the local level. The essential feature of a cellular automaton is that, despite the extreme simplicity of rules locally, globally its behaviour is impressively fertile and complex. By analogy with what happens in cellular

automata, researchers have carved up urban space into cells (in practice an area of some hectares). The conditions of each cell are described by variables typifying the diverse uses of spaces (residential, commercial, industrial, suitable for building, etc). The region will grow under the impact of changes affecting these variables and as a result of decisions made by different actors (population, entrepreneurs, administrators / public officers) with the support of a set of probabilistic rules. Concretely, for each cell researchers specify the surface area designated for the various purposes, the socio-economic variables (the inhabitants, the price of land according to the diverse target uses, rents, etc); they also specify the variables that represent the cell's relations to the urban and extra-urban communication network, etc. The cell evolves through concrete events, as for example the creation of new housing areas, vacating a flat or moving into one, or the conversion of some space from one kind of use to another. Once most variables have been fixed, mathematical simulations can be launched, so that on the basis of the few rules used we may witness the emergence of the complex structures that define and identify a city.

Can one tell whether the model is dependable?
Researchers use a mathematical tool called Master Equation (ME) which enables them to obtain a set of differential equations representing the dynamics of the urban system. By reducing a complex system to a list of numerical data, one can also confront more effectively the patterns created by the simulation with those observed in the region. This comparison, known technically as 'calibration', is crucial if one wants to invest the model with “divinatory” powers. The idea is to select a time span in the recent history of a city, to adjust the parameters of the model in such a way that the mathematical simulation may be a true replica of the development of the region as de facto observed. Once the calibration process is over, the mathematical model may be seen as an idealised version of the real urban system. At this point one may try to draw up general forecasts on the city's evolution.

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