Mehdi Farsi, Massimo Filippini, Michael Kuenzle

Unobserved heterogeneity in stochastic cost frontier models : a comparative analysis

Quaderno N. 03-11

Decanato della Facoltà di Scienze economiche
Via G. Buffi, 13 CH-6900 Lugano

UNOBSERVED HETEROGENEITY IN STOCHASTIC COST FRONTIER MODELS: A COMPARATIVE ANALYSIS*

Mehdi Farsi Massimo Filippini Michael Kuenzle

Department of Economics
University of Lugano
Via Ospedale 13, 6900 Lugano, Switzerland
and
Swiss Federal Institute of Technology
ETH Zentrum, WEC, 8092 Zurich, Switzerland

October 2003

∗ The authors are grateful to the Ticino's Dipartimento Sanità e Socialità for providing the data and their general support. We would also like to thank William Greene and Luca Crivelli for helpful comments and discussions, Ilaria Mosca and Chiara Gulfi for their excellent assistance and Giorgio Borradori for his help in understanding the data. We also benefited from helpful comments of the participants of the 8th European Workshop on Efficiency and Productivity Analysis (Ovideo, 2003). The views expressed in this paper are strictly personal. Responsibility for all remaining errors lies solely with the authors.

ABSTRACT

This paper studies a number of stochastic cost frontier models comparing their ability to distinguish unobserved heterogeneity from inefficiency variation among firms. The main focus is on the panel data models that incorporate firm-specific effects in a stochastic frontier framework, as proposed by Greene (2002, 2004). In cases where the unobserved heterogeneity is correlated with some of the explanatory variables, while the random effects estimators can be biased the fixed effects model may overestimate inefficiency scores. In line with Mundlak (1978), a simple method is proposed to include such correlations in random effects specification. The models are applied to a panel of 36 Swiss nursing homes operating from 1993 to 2001. The estimation results are compared and the resulted improvements are discussed. The results suggest that the proposed specification can avoid the inconsistency problem while keeping the inefficiency estimates unaffected.

1. Introduction

Following the work of Aigner, Lovell and Schmidt (1977), stochastic frontier models have been subject of a great body of literature resulting in a large number of econometric models to estimate cost and production functions. Kumbhakar and Lovell (2000) provide an extensive survey of this literature. One of the most important issues in these models is adjusting for the unobserved heterogeneity among firms functioning in different production environments. Individual firms face different external factors that could influence their production costs but are not under their control. These factors may be environmental such as network effects in network industries or related to output characteristics such as the severity of illness in the health sector and the demand fluctuations in electricity utilities. Some of these factors are observed and can be controlled for in the analysis. However, in many cases the data are not available for all these variables. Moreover, the relevant factors are often too complex to be quantified by simple indicators. For instance, factors such as the patient case-mix of a hospital and the network's shape of an electricity distribution company are hard to measure or require a great deal of information that is not usually available. Both these factors are generally beyond the firms' control but affect their costs significantly.

A stochastic frontier model by definition includes a random error term that captures the idiosyncratic heterogeneity among different observations. In panel data where an individual firm is observed several times, the firm-specific unobserved variations can also be taken into account through fixed or random effects. This is an important practical advantage because in many cases the relevant environmental factors are location characteristics that vary among firms but are constant over time. For instance the natural obstacles in a railway network such as high slopes or forest areas, or the average wealth of a community that may affect their health status, thus the operating costs of the neighboring hospital, are generally stable over a relatively long period of time.[1]

The first use of panel data models in stochastic frontier models goes back to Pitt and Lee (1981) who interpreted the panel data random effects as inefficiency rather than heterogeneity.[2] This tradition continued with Schmidt and Sickles (1984) who used a similar interpretation applied to a panel data model with fixed effects. Both models have been extensively used in the literature. A main shortcoming of these models is that any unobserved, time-invariant, firm-specific heterogeneity is considered as inefficiency. Thus an important advantage that panel data models can offer is overlooked. In more recent papers random effects model has been extended to include time-variant inefficiency. Cornwell, Schmidt and Sickles (1990) and Battese and Coelli (1992) are two important contributions in this regard. In particular the former paper proposes a flexible function of time with parameters varying among firms. However, in both these models firm-specific effects are considered as inefficiency. Another problem arises when the firm-specific effects are correlated with the explanatory variables.[3] In such cases, the random effects (RE) estimators are affected by heterogeneity bias,[4] but the fixed effects (FE) model while being consistent regarding the cost frontier slopes, usually overestimates efficiency variations. Therefore in many cases these models do not provide a unified approach for estimating cost frontier and inefficiencies. An exception is Cornwell et al. (1990)'s model which extends on Hausman and Taylor (1981)'s instrumental variable methodology. This model however requires the assumption that a sufficient number of explanatory variables are uncorrelated with random effects.

1 Note that most of the panel data used in the literature cover periods from 5 to 10 years.

2 Pitt and Lee (1981)'s model is different from the conventional RE model in that the individual-specific effects are assumed to follow a half-normal distribution. Important variations of this model were presented by Schmidt and Sickles (1984) who relaxed the distribution assumption and use the GLS estimator, and by Battese and Coelli (1988) who assumed a truncated normal distribution.

3 As we see later most of the relevant firm-specific factors are potentially correlated with some of the explanatory variables.

4 The term "heterogeneity bias" is used by Chamberlain (1982) to refer to the bias induced by the correlation between individual effects and explanatory variables in a general RE model.

A common feature of all these models is that they do not fully separate the sources of heterogeneity and inefficiency at the firm level. An alternative approach is to consider an additional stochastic term for cost efficiency. Theoretically, a stochastic frontier model in its original form (Aigner et al., 1977) can be extended to panel data models, by adding a fixed or random effect in the model. There are however few papers that have explored this possibility. The earliest attempt to use a panel data frontier model with firm dummies can probably be attributed to Polachek and Yoon (1996). Greene (2002a) discussed the numerical obstacles that have apparently delayed such a development. He proposed numerical solutions for both models with random and fixed effects, which he respectively refers to as "true" fixed and random effects models. In this paper we use the Greene's true RE model, which is basically the original cost frontier model with a random intercept.

This paper also proposes an alternative specification of RE models that controls for the correlation between firm-specific effects and explanatory variables. This model draws upon Mundlak (1978)'s formulation of a "within" estimator in the random effects framework. When applied to the conventional RE model, the resulted GLS estimator is identical to the FE estimator, thus unbiased. The inefficiency estimates are however adjusted for the correlation with exogenous variables. A similar method can be applied to the true RE model to decrease the heterogeneity bias.[5]

The main purpose of this paper is to study the extent to which these alternative models can improve the estimates of cost frontier and inefficiency scores. The models are estimated for a sample of 36 nursing homes operating in Ticino, the Italian-speaking region of Switzerland, over a nine-year period from 1993 to 2001. The alternative models are compared regarding their performance on the cost function slopes and inefficiency estimates. The conventional FE estimators of the cost function are assumed to be unbiased, thus used as a benchmark to which other models are compared.

The results suggest that as far as the heterogeneity bias is concerned, while the random constant frontier model (true RE) slightly improves the results, the proposed Mundlak adjustment brings the estimates very close to the unbiased estimators. As for the inefficiency scores, the estimates obtained from alternative models show a generally weak correlation. As expected, the FE model gives extremely high inefficiency values. Our analysis suggests that these values capture at least partially, the firms' heterogeneity that is correlated with

5 This argument is based on an analogy with a GLS model that can be transformed to a "within" estimator by using Mundlak's specification. However, it should be noted that given that the residual term in frontier models is asymmetric it is not clear whether this modification has the same effect in these models

exogenous variables. In fact, when these correlations are included in the model specification, the inefficiency estimates are systematically lower than comparable models. The results are in general promising in that the estimated cost frontier is similar to that of a conventional FE model thus unbiased, and the inefficiency estimates remain in a reasonable range. Our results also suggest that the average inefficiency scores and the their time trends are quite similar among comparable models with time-variant inefficiency.

The paper is organized as follows. Section 2 presents a brief and selective review of the existing literature. The model specification and the methodology are respectively described in sections 3 and 4. The data are explained in section 5. Section 6 presents the estimation results and section 7 concludes the paper.

2. Background

Heterogeneous production environments, which are not under the management's control, may influence the production process and incurred costs. These differences when observed or measured by observed proxies, can be incorporated in the estimation methods. A variety of methods exist in the stochastic frontier literature.[6] The focus of this paper is upon the cases in which such environment-related factors are not observed, but are constant for each firm. The basic panel data formulation, introduced by Schmidt and Sickles (1984), is a model in which the firm-specific stochastic term is interpreted as inefficiency. This term can be alternatively identified as a fixed intercept for each firm (FE model) or as an *iid* random term (RE model). In the case of a cost function, this model can be written as:

$$y_{it} = \mathbf{x}_{\mathbf{it}}'\boldsymbol{\beta} + \alpha_i + v_{it} \tag{1}$$

$$\hat{u}_i = \hat{\alpha}_i - \min_i(\hat{\alpha}_i) \geq 0. \tag{2}$$

where subscripts i and t identify the production unit and time respectively; y_{it} is a measure of costs (usually in logs); $\mathbf{x}_{\mathbf{it}}$ is a vector containing output quantities, input prices, and other exogenous variables; $\boldsymbol{\beta}$ is the parameter vector to be estimated; v_{it} represents an *iid* stochastic error and u_i is a nonnegative term representing cost inefficiency.

The fixed effects version of this model is estimated as a within estimator without any further distribution assumption on α_i. In particular, the fixed effects (α_i) can be correlated with the regressors ($\mathbf{x}_{\mathbf{it}}$). As for the random effects version, assuming that u_i is an *iid* stochastic term, the model can be estimated by Generalized Least Squares (GLS) method. In other variations of this model, the inefficiency component (u_i) is assumed to have a half-normal distribution (Pitt and Lee (1981)), or a truncated normal distribution (Battese and Coelli (1988)). A shortcoming of these models is that the inefficiency component is constant over time.[7] As discussed earlier Battese and Coelli (1992) among others, proposed alternative forms of deterministic variation of the inefficiency with time. However, as pointed out by Greene (2002b) in most of these models the time variation of efficiency terms is not stochastic and is assumed to follow a more or less restrictive form. Another drawback of these models is that any time invariant unobserved heterogeneity is pushed into the inefficiency component.

To overcome these problems, Greene (2002a) proposes a cost frontier model that includes firm dummies as explanatory variables (see also Polachek and Yoon (1996)):

$$y_{it} = \boldsymbol{\beta}'\mathbf{x}_{\mathbf{it}} + \alpha_i + v_{it} + u_{it}, \tag{3}$$

$$u_{it} \sim \left|N(0,\sigma_u^2)\right|, \quad v_{it} \sim N(0,\sigma_v^2). \tag{4}$$

This model is estimated by maximum likelihood method.[8] Greene refers to this extension as a “true” FE model to show the contrast with the FE framework commonly used in the frontier literature (Schmidt and Sickles (1984)). Unlike that model in which the fixed effects are interpreted as inefficiency, in Greene’s model the fixed effects represent the unobserved heterogeneity.

As in the standard FE model, the presence of the individual effects creates an incidental parameter problem, reducing the estimation efficiency. Moreover, since T is usually small, the estimates of individual effects (α_i) may incur large errors that can directly affect inefficiency estimates u_{it}. Using simulated samples, Greene (2004) shows that even in a short panel, although the fixed effects are largely biased, as far as the structural parameters (such as slopes of the cost function) and inefficiency scores are concerned, the model performs reasonably well.[9] However, Greene’s simulated samples are generated based upon the estimated parameters of a real sample of US banks in which the heterogeneity bias

6 These methods can be classified into two main categories: the models that include these observed factors directly in the cost function structure and those that consider them in the residual (inefficiency term). See Kumbhakar and Lovell (2000) for a survey.

7 Although this assumption may be reasonable in short panels, it is desirable to allow for time-varying inefficiency. Particularly, since the available data are commonly observed once a year it is conceivable that a firm’s managers modify their production plans from one year to another.

8 See Greene (2002a,b) for the derivation of the ML estimator.

9 While the bias on the firm dummies is about 300% or more (mostly toward zero), the average bias of inefficiency estimates is about 15%. The coefficients’ biases vary between 2 to 14 percent.

appears to be insignificant.[10] As we see later, the nursing home sample studied in this paper provides an example of strong unobserved heterogeneity that is potentially correlated with explanatory variables.

Greene (2004) also presents an extension to the standard RE model.[11] This extension is a random constant model, which is parameterized as:

$$y_{it} = \alpha_i + \boldsymbol{\beta}'\mathbf{x_{it}} + v_{it} + u_{it}, \tag{5}$$

where α_i is a time invariant, firm specific random term which should capture the firm specific heterogeneity and the other variables are according to (4). Following Greene, this model is hereafter referred to as "true" random effects model. The difference with the true FE model (3) is that the heterogeneity term (α_i) is assumed to have an *iid* normal distribution.

As this model appears to have three different disturbances, one could raise the question of identification. However, as Greene (2002a) argues, the composite error term $\varepsilon_{it} = u_{it} + v_{it}$ can be seen as a single stochastic term with an asymmetric mixed distribution with a known density.[12] The conditional likelihood function $f(y_{it}|\alpha_i)$ can thus be readily derived. However, since the unconditional likelihood function does not have a closed form solution, Greene (2002a) proposes Maximum Simulated Likelihood Estimation (MSLE) method, namely by integrating out α_i using Monte Carlo method.

In the RE framework, it is assumed that the firm-specific effects are uncorrelated with the explanatory variables in the model. Therefore, all the extensions of the RE model are prone to heterogeneity bias due to such correlation. However, the refinement of the model to separate different sources of heterogeneity may improve the performance of the model, especially regarding the inefficiency estimates. One can expect that the sensitivity of the results to the adopted model is directly related to the extent to which the firm-specific heterogeneity is correlated with the explanatory variables. In order to study this issue, we turn to the application of these alternative models to a nursing home sample in which the heterogeneity among firms is clearly correlated with the explanatory variables. In order to avoid the estimation errors in the true FE model, an alternative specification of the true RE model based on Mundlak's formulation is proposed.

---

10 In the US banks sample the Hausman test does not reject the hypothesis of no correlation between individual effects and explanatory variables. The Chi-square value with 10 degrees of freedom is only 7.47. See Greene (2002a) page 36.
11 See also Greene (2002b).
12 See Aigner et al. (1977) for the density function of a mixed normal-truncated-normal distribution.

3. Model specification

A nursing home can be approximately represented as a production unit transforming labor and capital services into patient-days of residential health and social care for elderly people.[13] Assuming that output level and input prices are exogenous, and that (for a given technology) firms choose input levels to minimize costs, the firm's total cost of operating a nursing home can be defined as a function of input prices and output. Moreover, in the cost model specification we take into account a number of output characteristics, which should capture, at least partially, the heterogeneity and quality dimensions of the nursing home's output. Costs can also vary with a time trend. The total cost frontier can therefore be represented by the following cost function[14]:

$$TC=f\,(Y, Q, R, P_K\,, P_L\,, \tau\,) \tag{6}$$

where TC is the total annual cost and Y is the output represented by the total number of resident-days of the nursing home. P_K and P_L are respectively the prices of capital and labor. Q represents the average dependency index calculated annually by the Regional Department of Public Health. This index measures the average required assistance of a given nursing home's patients with normal daily activities such as eating, personal care or performing physiological functions. Q varies from 1 to 3, with 3 representing the most severe (dependent) case. R is the nursing staff ratio, that is the ratio of the number of employed nurses in a nursing home to the number of nurses that should be employed according to the guidelines of the Regional Department of Public Health.[15] Since the nursing care is a labor-intensive service and the quality of care depends on the time spent by nurses for each patient, this variable represents the quality of output and the production process.[16] Finally, τ is a linear time trend that captures the changes in technical efficiency associated with technical progress.

It is generally assumed that the cost function given in (6) is the result of cost minimization given input prices and output and should therefore satisfy certain properties.[17] Mainly, this function must be non-decreasing, concave, linearly homogeneous in input prices and non-decreasing in output. To estimate the cost function (6), a translog functional form is employed. This flexible functional form is a local, second-order approximation to any

---

13 In Switzerland, in addition to the usual nursing care, nursing homes also provide basic medical services to their residents.

14 For a similar cost model specification see Filippini (2001).

15 These guidelines are only recommendations and the nursing homes are not required to exactly follow them.

16 See Cohen and Spector (1996) and McKay (1988) for a similar approach in cost model specification for nursing homes. Cohen and Spector measured quality of care by staff to resident ratios. McKay used "nursing hours per patient" to measure the nursing home's quality.

arbitrary cost function. It places no *a priori* restrictions on the elasticity of substitution and allows the economies of scale to vary with the output level. The translog approximation to (6) can be written as:

$$\begin{aligned}\ln\left(\frac{TC_{it}}{P_{K_{it}}}\right) = {} & \alpha_0 + \alpha_Y \ln Y_{it} + \alpha_Q \ln Q_{it} + \alpha_R \ln R_{it} + \alpha_L \ln \frac{P_{L_{it}}}{P_{K_{it}}} \\ & + \frac{1}{2}\alpha_{YY}(\ln Y_{it})^2 + \frac{1}{2}\alpha_{LL}\left(\ln \frac{P_{L_{it}}}{P_{K_{it}}}\right)^2 + \frac{1}{2}\alpha_{QQ}(\ln Q_{it})^2 + \frac{1}{2}\alpha_{RR}(\ln R_{it})^2 \\ & + \alpha_{YL} \ln Y_{it} \ln \frac{P_{L_{it}}}{P_{K_{it}}} + \alpha_{YQ} \ln Y_{it} \ln Q_{it} + \alpha_{YR} \ln Y_{it} \ln R_{it} + \alpha_{QR} \ln Q_{it} \ln R_{it} \\ & + \alpha_{LQ} \ln \frac{P_{L_{it}}}{P_{K_{it}}} \ln Q_{it} + \alpha_{LR} \ln \frac{P_{L_{it}}}{P_{K_{it}}} \ln R_{it} + \alpha_\tau \tau + \alpha_i + \varepsilon_{it}\end{aligned} \qquad (7)$$

with $i = 1, 2, ..., N$ *and* $t = 1,2,...,T$

where subscripts *i* and *t* denote the nursing home and year respectively, α_i is a firm-specific effect and ε_{it} is an *iid* error term which can be symmetric or asymmetric dependent upon the adopted econometric model. The models used in this paper are based on two general frameworks: Schmidt and Sickles (1984)'s model that assumes a symmetric ε_{it} (without any further distribution assumption), and Aigner et al. (1977)'s original framework in which ε_{it} is assumed to have a composite normal-half-normal distribution.

All variables are normalized by the corresponding sample medians. Therefore, the translog form is considered as a second order approximation around the sample median.[18] As it can be seen in equation (7), linear homogeneity in input prices is imposed by dividing total costs and input prices by capital price. The other theoretical restrictions are verified after the estimation. In particular, the concavity of the estimated cost function reflects the fact that the cost function is a result of cost minimization. However, this assumption may be unrealistic in non-profit firms. In such cases, the functions based on cost optimization may still be used as "behavioral" cost functions and can be helpful in studying the behavior of such firms.[19] Especially, since all the nursing homes in our sample are non-profit, it can be reasonably assumed that they follow (or should follow) a similar objective function, implicitly set by the regulators. Given this assumption comparing costs among different firms can indicate which firms achieve these objectives with less costs.

---

17 For more details on the functional form of the cost function see Cornes (1992), p.106.

18 Translog functional form requires that the underlying cost function be approximated around a specific point. In our case this point is taken as the sample median. We choose the median rather than the mean, because it is less affected by outliers and thus the translog approximation will have a better precision.

Input prices and output are assumed to be exogenous, thus beyond the firm's control. In a regulated industry these conditions are generally satisfied. Ticino's non-profit nursing homes are fully regulated by the canton's government. The residents are assigned to nursing homes by their community's authorities, mainly based on their location, and the nursing homes' costs are refunded on a cost-plus basis.

4. Methodology

As we discussed earlier, in RE models the unobserved firm-specific heterogeneity is assumed to be uncorrelated with the included explanatory variables in the model. The estimates are biased (heterogeneity bias) if this condition does not hold. On the other hand, the FE models that do not impose any correlation assumption may result in high estimation errors in inefficiency measures. In order to overcome this problem, we use Mundlak (1978)'s formulation. In our knowledge this is the first time that Mundlak's approach is used in cost frontier analysis. Mundlak proposes a modified random effects model, in which the correlation of firm-specific effects with explanatory variables are considered in an auxiliary regression given by:

$$\alpha_i = \gamma \bar{X}_i + \delta_i \qquad \bar{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}\ ,\ \ \delta_i \sim iid(0,\sigma_\delta^2) \tag{8}$$

where X_{it} is the vector of all explanatory variables and γ is the corresponding vector of coefficients. Equation (8) actually divides the firm-specific stochastic term into two components: The first part can be explained by exogenous variables, whereas the remaining component (δ_i) is orthogonal to explanatory variables. If the inefficiency is assumed to be constant over time, this part can be interpreted as the firm's inefficiency. In this case, inefficiencies can be estimated by comparing each firm to the firm with the minimum δ_i, that is: $\hat{\delta}_i - \min\{\hat{\delta}_i\}$.

Equation (8) can be readily incorporated in the main regression equation (7). In the special case in which the error term (ε_{it}) is symmetric, the GLS estimators of the resulting equation are identical to the FE estimators of the original equation (within estimators), thus unbiased.[20] In the general case where ε_{it} is a composite asymmetric term, since the correlation between individual effects and explanatory variables is at least partly captured in the model, the heterogeneity bias is also expected to be minimal. Therefore, one can expect the proposed

19 See Bös (1986), page 343.
20 See also Hsiao (2003), pp. 44-46, for a proof of the identity of Mundlak's GLS estimators and FE estimators.

specification can avoid the heterogeneity bias and at the same time gives reasonable estimates of inefficiency. Moreover, time-invariant factors can also be included in the RE model.

The heterogeneity bias problem and its effect on inefficiency estimates are studied by a comparative analysis of six different models. All models are based on the specification given in equation (7). The differences are related to the assumptions imposed on stochastic components α_i and ε_{it}. Table 1 summarizes the six models used in the paper. The first model is a fixed effects model. In this model the firm-specific effects are estimated as constant numbers, thus can be correlated with the explanatory variables. As is well known in the literature, the FE or "within" estimators are not influenced by heterogeneity bias.[21] In the cost frontier literature the inefficiency scores are estimated as the distance from the firm with the minimum estimated fixed effect, that is $\hat{\alpha}_i - \min\{\hat{\alpha}_i\}$, as proposed by Schmidt and Sickles (1984).

Model *II* is a random effects model, which is estimated using the GLS method. The inefficiency term is estimated following the approach proposed by Schmidt and Sickles (1984). The important limitation of this model is the assumption that the firm-specific stochastic term α_i, here assumed to be the firm's inefficiency, is uncorrelated with the explanatory variables. Although it is reasonable to assume that the firm's cost-inefficiency[22] is not correlated with exogenous variables, the firm-specific stochastic term may contain other unobserved environmental factors, which may be correlated with explanatory variables.

In both models (*I* and *II*), inefficiency indicators may include unobserved environmental factors, thus may overstate the firms' inefficiency. There are however two factors that may exacerbate this problem in the FE model. First, unlike the RE model, the firm-specific effects do not follow a single distribution, thus can have a relatively wide range of variation. Secondly, these effects can be correlated with the explanatory variables, thus can also capture the heterogeneity factors that are correlated with the regressors. Whereas in the RE model in which the firm-specific effects are by construction uncorrelated with the regressors, these factors are suppressed at least partially through the "between" variations, into the regression coefficients. Model *III* is the GLS model specified in line with Mundlak's formulation. As discussed earlier, the cost function coefficients are identical to model *I*, but the inefficiency estimates are adjusted for correlation with exogenous variables.

Model *IV* is a pooled frontier model in which the firm-specific effect is assumed to be zero. Thus the sample is considered as a series of cross sectional sub-samples pooled together.

21 See Baltagi (2001) for an extensive discussion.

The random error term is divided into two components: a normal error term v_{it}, capturing heterogeneity and a half-normal random term u_{it}, representing the inefficiency as a one-sided non-negative disturbance. This model is based on the original cost frontier model proposed by Aigner et al. (1977). The firm's inefficiency is estimated using the conditional mean of the inefficiency term $\mathrm{E}\left[u_{it} \mid u_{it}+v_{it}\right]$, proposed by Jondrow et al. (1982).

Table 1. Econometric specifications of the stochastic cost frontier

	Model I **FE**	*Model II* **RE (GLS)**	*Model III* **RE (GLS) with Mundlak formulation**	*Model IV* **Pooled**	*Model V* **True RE**	*Model VI* **True RE with Mundlak formulation**
Firm-specific component α_i	fixed	*iid* $(0, \sigma_\alpha^2)$	$\alpha_i = \gamma \bar{X}_i + \delta_i$ $\bar{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}$ $\delta_i \sim iid(0, \sigma_\delta^2)$	none	*iid* $(0, \sigma_\alpha^2)$	$\alpha_i = \gamma \bar{X}_i + \delta_i$ $\bar{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}$ $\delta_i \sim iid(0, \sigma_\delta^2)$
Random error ε_{it}	*iid* $(0, \sigma_\varepsilon^2)$	*iid* $(0, \sigma_\varepsilon^2)$	*iid* $(0, \sigma_\varepsilon^2)$	$\varepsilon_{it} = u_{it} + v_{it}$ $u_{it} \sim N^+(0, \sigma_u^2)$ $v_{it} \sim N(0, \sigma_u^2)$	$\varepsilon_{it} = u_{it} + v_{it}$ $u_{it} \sim N^+(0, \sigma_u^2)$ $v_{it} \sim N(0, \sigma_u^2)$	$\varepsilon_{it} = u_{it} + v_{it}$ $u_{it} \sim N^+(0, \sigma_u^2)$ $v_{it} \sim N(0, \sigma_u^2)$
Inefficiency	$\hat{\alpha}_i - \min\{\hat{\alpha}_i\}$	$\hat{\alpha}_i - \min\{\hat{\alpha}_i\}$	$\hat{\delta}_i - \min\{\hat{\delta}_i\}$	$\mathrm{E}\left[u_{it} \mid u_{it}+v_{it}\right]$	$\mathrm{E}\left[u_{it} \mid u_{it}+v_{it}\right]$	$\mathrm{E}\left[u_{it} \mid u_{it}+v_{it}\right]$

Models *V* and *VI* are extensions to model *IV* in that they include an additional firm-specific effect (α_i) to represent the unobserved heterogeneity among firms. In both models this effect is considered as a random effect. Model *V* is based on true RE model proposed by Greene (2002a,b).[23] Finally model *VI* is the true RE model modified by Mundlak's specification. This model not only includes a firm-level source of heterogeneity, potentially correlated with explanatory variables, it also allows for a time-variant inefficiency term.

In our comparative analysis we consider two aspects of the models' performance. The first dimension is the estimation of the cost function's coefficients. In cases such as nursing homes (or in general health services), where the costs are affected by the case-mix severity, a number of location-related factors can affect both costs and explanatory variables. For instance, larger nursing homes are usually located in more populated urban areas where patients might be sicker (thus more costly) and the price of labor is potentially higher. Such

22 Note that here the cost-efficiency does not include scale efficiency.
23 This model is a special case of a stochastic frontier model with random parameters (in this case random intercept).

relationships imply a positive correlation between the output level and labor price with the case-mix severity, which is not fully captured by the included factors in the model.[24] The Hausman test is used to confirm that the firm-specific effects are correlated with the explanatory variables. In this case the FE estimators are unbiased, thus provide a benchmark to which other models can be compared. On the other hand, the GLS estimators are biased and therefore provide an indication for the direction of heterogeneity bias. Noting that the Hausman test statistics is based on an overall distance between the two estimators, for each model we compare the estimated cost function's coefficients with the corresponding estimates from the FE and GLS models (models *I* and *II* here).

The heterogeneity bias is expected to be relatively low in models *III* and *VI* that directly control for correlation between individual effects and explanatory variables. In other models there is no way to predict the bias. One can argue that models with more general error structures have lower biases because the residuals can capture a larger part of the correlations between unobserved heterogeneity and explanatory variables, thus leaving the coefficients less affected. However, the residuals are by definition uncorrelated with explanatory variables and the extent to which they may confound such correlations with errors may significantly vary from one sample to another. Especially since the frontier estimators are non-linear, the prediction of the biases is not straightforward. This theoretical discussion is beyond the scope of this paper. Here we rather focus on the evaluation of the models with respect to our sample.

The second aspect of the models' performance concerns the estimation of inefficiency scores. Since they are based on certain interpretation of the stochastic terms included in the model, the inefficiency estimates are considered as a separate dimension of the model's performance. In fact, an unbiased estimation of the cost function is a necessary but not sufficient condition for consistent estimation of inefficiency. In the first three models (*I*, *II* and *III*), the firm's inefficiency is assumed to be constant over time, thus captured by the firm-specific effects. In models *IV*, *V* and *VI* on the other hand, the firm's inefficiency can vary from one year to another. In these models, the skewed stochastic error term is interpreted as inefficiency. Except for the FE model (model *I*), in all these models it is assumed that the

24 The average dependency index, which is included in the model, only measures the time required for nursing care, thus captures only one aspect of case-mix severity. Other factors like the need for medical treatment and drugs are not observed.

firm's cost efficiency is not correlated with the explanatory variables.[25] This assumption is consistent with the requirement that the explanatory variables are exogenous.

The FE formulation in model *I* has two important limitations. First, the time invariant variables are captured by the fixed effects and cannot be included in the model. This implies that the inefficiency estimators include the variations in time-invariant firm characteristics.[26] Moreover, the estimated fixed effects include unobservable firm-specific factors that are correlated with explanatory variables. However, as is common in most frontier models, the firm's inefficiency *per se* is not correlated with exogenous variables like output and input prices. Therefore, in cases where unobserved environmental factors are likely to affect costs, model *I* appears to be inadequate regarding the estimation of inefficiencies. Model *II* is expected to have a better performance because the individual effects are by construction uncorrelated with explanatory variables, thus less affected by exogenous variables. However, the inefficiency estimates may still contain firm-specific heterogeneity that is not related to inefficiency. The Mundlak's adjustment used in model *III* should take care of such heterogeneity to the extent that it is correlated with the explanatory variables.

In models *IV*, *V* and *VI*, where the inefficiency can vary with time, one could expect to have higher inefficiency estimates compared to models *II* and *III*. Model *IV* ignores the firm-specific heterogeneity, thus may overestimate inefficiency compared to models *V* and *VI*. In model *VI*, Mundlak's adjustment may help to completely separate the correlation effects, thus leads to lower estimates of inefficiency.

Except for the FE model (Model *II*) that, for reasons mentioned above, is expected to have a poor performance, there is no general, clear-cut distinction among the studied models regarding their performance as to efficiency estimation. Rather, each model implies a different interpretation of cost-inefficiency. If the inefficiency is believed to be persistent, the models with time-invariant inefficiency may be more relevant. GLS models (*II* and *III*) while being free of additional distribution assumptions on inefficiencies, because of the symmetry of random effects, implicitly assume that only one firm is fully efficient and all other firms are in fact more or less inefficient. On the other hand, the frontier models with half-normal inefficiency, or any other asymmetric distribution with zero (or close to zero) mode, assume that most of the firms are likely to be efficient and the probability of being inefficient is

25 It is worth noting that here cost inefficiency is defined as the excess costs due to the firm's technical problems or to suboptimal allocation of resources. Other inefficiency sources like scale inefficiencies, which are beyond the firm's control are excluded.

26 As our specification does not include any time-invariant factor, this statement does not apply here.

decreasing with the degree of inefficiency. We contend that in most regulation applications, the latter assumption is more consistent with the real world as well as economic theory.

Most frontier models assume that inefficiency is uncorrelated with explanatory variables included in the cost function.[27] While being practical for estimation purposes, this assumption can be justified based on the fact that the apparent excess costs that are correlated with explanatory variables may be due to factors beyond the firm's control. To the extent that the firm's inefficiency is not correlated with the explanatory variables Mundlak's adjustment is likely to improve the estimations. The purpose of this paper is not to identify the most appropriate method, which could differ from one case to another. Rather, our comparative analysis should highlight in each one of the models, the relation of what is called inefficiency with other sources of heterogeneity as well as with the explanatory variables. In any case, a high correlation between the inefficiency estimates can be inferred as an indication of robustness and validity of individual approaches. Therefore, the correlation between the inefficiency scores estimated by different models is studied. Different models are also compared with respect to the average inefficiency of the whole sample. In cases where the inefficiency varies over time, the annual averages are also compared.

5. Data

The data set used in this paper is prepared based on the annual accounting reports of 36 non-profit nursing homes in Ticino, the Italian-speaking region of Switzerland, over the 9-year period from 1993 to 2001. The sample includes more than two thirds of Ticino's nursing homes. All the nursing homes in the sample provide inpatient services.[28] There are four missing observations in 1993, leaving a total of 320 observations. The variables include total costs, total number of employees (in terms of full-time equivalent units), average wage per employee per year, total number of beds and total number of resident-days. Other characteristics are the average dependency grade of the residents and the number of caring personnel working for the nursing home.

Total cost is taken as the total annual expenditures of the nursing home. Output is measured as the total number of patient-days of the nursing home. Average yearly wage rates are estimated as the weighted mean of the average wage rates of different professional

---

27 The only exception is the FE models that interpret the effects as inefficiency.

28 There are some nursing homes that offer the possibility of nursing care in external residential apartments. The nursing care of this type is less intensive (thus less costly) than the care given to the home's residents. For this reason we excluded four nursing homes whose share of external beds is more than 10 percent of their total

categories working in a nursing home, including nurses, administrative and technical staff and physicians. Following Friedlaender and Wang Chiang (1983) and Filippini (2001), the capital price is calculated from the residual costs divided by the capital stock. Residual cost is total cost minus labor cost. Similar to Wagstaff (1989), the capital stock is approximated by the number of beds operated by the nursing home.[29] The quality indicators, Q and R, (as described earlier) are calculated annually by the regional Department of Public Health.

Table 1. Descriptive statistics (320 observations)

	Mean	Standard Deviation	Median	Min.	Max.	Fraction of between variation*
Total annual costs per resident-day	184.05	28.92	183.10	111.85	279.81	.307
Total annual resident-days (Y)	23,176	9684	21,482	6,525	58,324	.848
Number of beds	66.23	26.81	61	28	162	.850
Average labor price (P_L) per employee per year	70,157	6,586	70,280	29,744	122,950	.099
Average capital price (P_K) per bed	11,008	2,579	10,714	3,466	22,426	.606
Average dependency index (Q)	2.575	.219	2.6	1.87	3	.387
Nursing staff ratio (R)	.963	.124	.97	.49	1.55	.235

* Fraction of variance due to between variation is defined as $Var(u_i)/\left(Var(u_i)+Var(\varepsilon_{it})\right)$, where u_i and ε_{it} are the residuals of a GLS regression of the corresponding variable on a constant. $i= 1, 2, .., N$ $t= 1,2,...,T$.

- All monetary values are in 2000 Swiss Francs (CHF), adjusted for inflation by Switzerland's global consumer price index.

The summary statistics of the main variables used in the analysis are given in table 1. As it can be seen in the table, there is a high variation in the costs of a patient-day care. The input prices show a great amount of variation as well. Part of these variations is associated with time variation. For instance the average cost of a patient-day care has increased from

beds. In our final sample there remain two nursing homes that offered external care (less than 10 percent) for some years during the study period.

29 A more precise estimation of capital stock would require capital inventory data, which are not available to us.

about 154 Francs in 1993 to 214 Francs in 2001. In the same period, the price of labor has increased about 15 percent in real terms and our measure of real capital price has increased about 20 percent. The last column of table 1 lists the fraction of the variance of each variable due to the sample's variation between different nursing homes. These numbers suggest that all the variables show significant variations both within and between nursing homes. This result justifies the use of panel data models, especially the FE estimator that relies upon "within" variations.

6. Estimation results

The estimated parameters of the basic cost frontier models are listed in table 2. The regression results show that all the first-order terms are significant and in a reasonable direction. As expected, output and prices have a positive effect on costs, and the nursing homes with a more severe case-mix and/or with a higher quality of service are relatively more costly.[30] Since total costs and the regressors are in logarithms and normalized by their medians, the first order coefficients are interpretable as cost elasticities evaluated at the sample median. The output elasticity is positive and implies that an increase in the supply will increase total cost. The results indicate unexploited scale economies in the production. Different models lead however to different results. A one percent increase in the number of patient-days of nursing home care will increase the total cost by about 0.75% to 0.92%. Other coefficients are also significantly different across different models.

Cost elasticities with respect to the output characteristics variables, Q and R, are positive and imply that an increase in the average required assistance of a home's patients or an increase in the ratio of the number of nurses employed by a nursing home and the number of nurses that should theoretically be employed will increase total cost. The coefficient of the linear trend suggests that the total costs have increased over time with a rate of about 0.9 to 1.8 percent per year. The growth of costs is a commonly observed phenomenon in labor-intensive industries such as health care, which usually face a persistent growth of labor price. The estimated cost functions do not however satisfy the concavity condition in input prices.[31] This may suggest that the estimated cost functions are not resulted from a completely unconstrained cost-minimization strategy. Namely, the firms' strategies are not responsive to

30 These findings are in line with the results obtained by Filippini (2001) using a shorter panel and a slightly different number of nursing homes.

31 Our results indicate that the Hessian matrix of the estimated cost function with respect to input prices (labor and capital) is not negative semi-definite, thus the concavity condition is not satisfied in any of the specifications.

changes in input factor prices. This can be explained by the fact that the input choices in Switzerland's nursing homes are rather constrained by regulation.[32]

Table 2. Estimated coefficients

	Model I **FE**	*Model II* **RE (GLS)**	*Model IV* **Pooled**	*Model V* **True RE**
α_Y	.750* (.028)	.890* (.017)	.925* (.014)	.869* (.007)
α_Q	.308* (.097)	.555* (.083)	.713* (.082)	.481* (.036)
α_R	.317* (.046)	.382* (.046)	.435* (.045)	.350* (.021)
α_L	.804* (.027)	.832* (.025)	.877* (.023)	.819* (.012)
α_{YY}	-.149* (.061)	-.024 (.053)	.050 (.043)	-.085* (.022)
α_{QQ}	-1.036 (.91)	-.558 (.90)	-.034 (.96)	-.440 (.52)
α_{LL}	.512* (.076)	.612* (.075)	.573* (.061)	.579* (.034)
α_{YQ}	.078 (.12)	-.011 (.12)	.051 (.14)	-.001 (.056)
α_{YL}	.004 (.045)	-.00006 (.042)	.050 (.036)	-.020 (.019)
α_{LQ}	.187 (.17)	.034 (.17)	.022 (.19)	.094 (.10)
α_{RR}	-.200 (.20)	-.113 (.21)	-.304 (.193)	-.176 (.09)
α_{YR}	.273* (.097)	.223* (.098)	.167 (.116)	.193* (.047)
α_{LR}	.395* (.12)	.348* (.12)	.408* (.12)	.412* (.045)
α_{QR}	-.187 (.34)	-.587 (.34)	-.740* (.356)	-.447* (.16)
α_τ	.018* (.002)	.012* (.002)	.009* (.002)	.014* (.001)
Constant	_	15.15* (.013)	15.10* (.014)	15.10* (.005)
R-square	.987	.975	_	_

- Standard errors are given in brackets. * means significant at less than 5%.
- The sample includes 320 observations (36 nursing homes).

The main observation on the results listed in table 2, is that the FE estimators (model *I*) can be singled out as extreme values for almost all the coefficients. The Hausman test

32 See Farsi and Filippini (2003) for a more detailed discussion.

rejects the hypothesis of no correlation between random effects and the explanatory variables quite significantly (Chi-square of 57.3 with 15 degrees of freedom). The FE model results are therefore unbiased and can serve as a benchmark for our estimations. This implies the inconsistency of all other models.

Table 3. Mundlak's formulation

	Model III **RE (GLS) with Mundlak formulation**		*Model VI* **True RE with Mundlak formulation**	
	Main Equation Coefficient	*Auxiliary Equation Coefficient*	*Main Equation Coefficient*	*Auxiliary Equation Coefficient*
α_Y	.750* (.028)	.184* (.041)	.762* (.016)	.175* (.019)
α_Q	.303* (.098)	.583* (.184)	.341* (.051)	.564* (.071)
α_R	.316* (.046)	.237 (.193)	.331* (.027)	.203* (.070)
α_L	.804* (.028)	.082 (.062)	.802* (.016)	.096* (.024)
α_{YY}	-.149* (.061)	.304* (.126)	-.142* (.033)	.286* (.050)
α_{QQ}	-1.048 (.91)	3.95 (3.76)	-.876 (.63)	2.72* (1.37)
α_{LL}	.513* (.077)	-.188 (.339)	.515* (.044)	-.173 (.13)
α_{YQ}	.077 (.12)	.436 (.592)	.051 (.066)	.471* (.212)
α_{YL}	.004 (.045)	.175 (.141)	-.012 (.021)	.191* (.052)
α_{LQ}	.187 (.17)	-.753 (.819)	.203 (.12)	-.780* (.303)
α_{RR}	-.201 (.20)	-.806 (1.25)	-.211 (.11)	-.568 (.44)
α_{YR}	.273* (.097)	.0011 (.35)	.240* (.054)	-.020 (.13)
α_{LR}	.395* (.12)	.289 (.531)	.365* (.060)	.402* (.19)
α_{QR}	-.185 (.34)	-1.53 (1.45)	-.242 (.22)	-1.38* (0.55)
α_τ	.018* (.002)	_	.018* (.001)	_
Constant	15.12* (.014)	_	15.08* (.007)	_
R-square	.982		_	

- Standard errors are given in brackets. * means significant at less than 5%.
- The sample includes 320 observations (36 nursing homes).

Table 3 lists the estimation results obtained from Mundlak's formulation. As expected, when applied to the RE (GLS) model, the main equation coefficients are quite close if not identical, to the within estimators in model *I*. Some of the auxiliary equation coefficients are significant indicating that the random effects are actually correlated with some of the explanatory variables. In particular the coefficients of output (*Y*) and the dependency ratio (*Q*) are highly significant and positive. This may suggest that the unobserved heterogeneity among nursing homes is partly due to the patients' unobserved severity characteristics. As argued earlier such characteristics may well be positively correlated with the nursing home's size. The last two columns of the table present the results of Mundlak's specification applied to the true RE model. Most auxiliary coefficients are significant, confirming high correlation of random effects with explanatory variables. Interestingly, Mundlak adjustment has a similar effect on the true RE model, bringing the estimated coefficients closer to the unbiased results of model *I*.

The first-order coefficients obtained from different models are compared in figure 1. This figure plots the ratio of the estimated coefficients to the corresponding estimates obtained from the FE model. For any given coefficient the distance from unity indicates the "distance" of the estimator from a consistent estimate of that coefficient. As it is seen in the figure, the pooled model's estimates are in general located relatively far from the FE estimates, suggesting that this model has the poorest performance with regard to heterogeneity bias. This is consistent with the fact that this model does not distinguish individual firms and may be strongly affected by the omitted variables bias. The coefficients estimated by the true RE model on the other hand, lie almost without exception, between those of the GLS model and the FE estimators. This result suggests that in our sample, compared to GLS, this model is less affected by heterogeneity bias. Finally, the estimates obtained from model *VI* are quite close to the unbiased estimators, suggesting that controlling for correlations in the true RE model can decrease the heterogeneity biases.

Figure 1
Ratio of different models estimators with respect to the corresponding fixed-effects model estimates

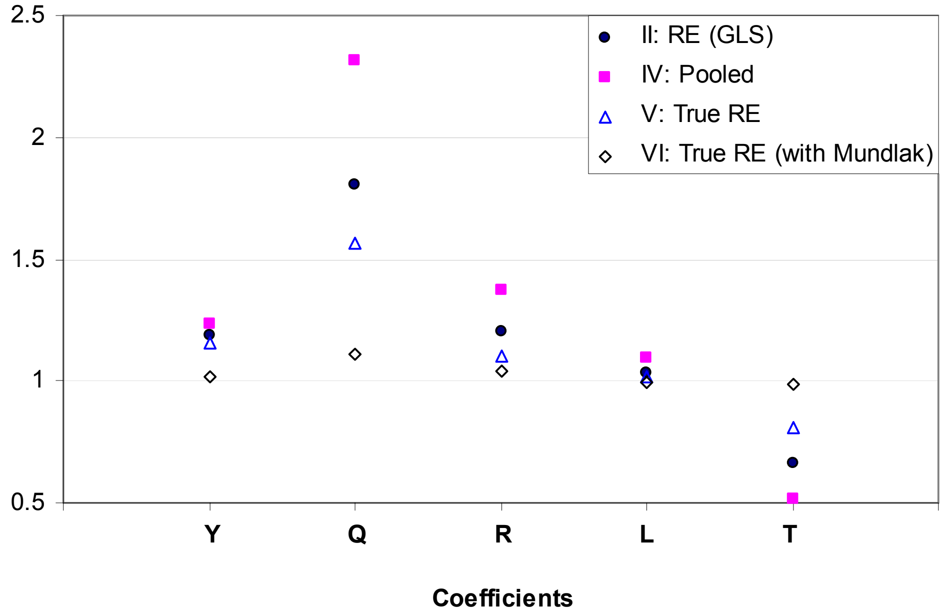


Table 4 provides a summary of the estimated inefficiency measures using different models. The inefficiency scores are taken equal to the inefficiency scores (u_{it}), obtained from the regression model. These measures represent the relative excess cost of a nursing home compared to a minimum level that would have been achieved had the firm operated as cost-efficient as the "best practice" observed in the sample. Note that in the first three models (*I*, *II* and *III*) inefficiency is assumed to be constant over time with a single fully efficient firm, while in models *IV*, *V* and *VI*, the firm's inefficiency is time-variant and most of the firms are expected to be fully efficient or close. Therefore, comparing the values across two groups should be done with caution.

As expected, the FE model predicts excessive inefficiency estimates averaging about 19% and up to a maximum of 38%. Model *II*'s results are less than half of these values, suggesting that the estimates in model *I* are confounding heterogeneity with inefficiency. This result suggests that both models are affected by the heterogeneity bias; while in the RE model, the coefficients capture most of the biases, in the FE model the bias appears only in the individual effects. The results also show that Mundlak's adjustment in model *III* improves the results in that while keeping the coefficients unbiased, it decreases the bias in inefficiency estimates by separating the correlation effects. As seen in table 4, compared to the GLS model, the inefficiency estimates are on average about 40% lower when these correlations are taken into account.

Table 4. Inefficiency measures:

	Model I **FE**	*Model II* **RE (GLS)**	*Model III* **GLS with Mundlak fromulation**	*Model IV* **Pooled**	*Model V* **True RE**	*Model VI* **True RE with Mundlak**
Mean	.191	.082	.050	.059	.051	.045
Median	.203	.089	.052	.054	.043	.040
Maximum	.379	.152	.104	.279	.251	.210
Minimum	0	0	0	.009	.006	.008
N	36	36	36	320	320	320

- Inefficiency measures represent the relative difference of a nursing home's actual costs to minimum costs from the best practice in the sample.

Comparing models with time-variant inefficiency (last three columns of the table) shows that the inefficiency estimates are on average more or less similar. This implies that in these models, inefficiency estimates are not much sensitive to the specification of firm-specific heterogeneity. The differences however, point to a similar pattern in that a better control for firm-specific heterogeneity decreases the inefficiency estimates. Namely, the average inefficiency score decreases from 5.9% in the pooled model (model *IV*) to 5.1% after controlling for firm-specific heterogeneity (model *V*), and to 4.5% with an additional Mundlak correction (model *VI*).

The pair-wise correlation coefficients between the inefficiency scores obtained from different models are presented in table 5. In order for the correlation coefficients to be comparable, they are calculated at the firm level using 36 observations (one observation for each firm). Namely, in models with time-variant efficiency, the inefficiency score is calculated as the firm's average inefficiency score over the sample period. Although there is no clear threshold to evaluate these correlations, we consider that a coefficient less than 0.9 is indicative of quite significant differences in both individual scores and ranks across the models. According to this criterion, the correlations between the models in each group (time-variant and time-invariant) are rather weak.

Table 5. Pair-wise correlation between inefficiency estimates from different models:

	Model I **FE**	*Model II* **RE (GLS)**	*Model III* **GLS with Mundlak fromulation**	*Model IV* **Pooled**	*Model V* **True RE**	*Model VI* **True RE with Mundlak**
Model I	1					
Model II	.849	1				
Model III	.343	.670	1			
Model IV	.534	.854	.834	1		
Model V	.888	.939	.555	.806 .902*	1	
Model VI	.260	.601	.941	.878 .901*	.546 .805*	1

- In models *IV*, *V* and *VI* the inefficiency estimates are the average values over the sample period. Correlation coefficients based on 320 observations are marked by an asterisk (*).

This result is in contrast with the results reported by Greene (2002a) who applied a series of alternative models to a short panel of US banks sample (*T*=5). In that analysis the inefficiency estimates obtained from Pitt and Lee (1981)'s model and a standard FE model (as in Schmidt and Sickles (1984)), both with time-invariant inefficiency, are very close. Similarly, there is a quite high correlation between the estimates from the true FE and true RE models, with time-variant inefficiency. Greene's results can however be explained by the fact that as suggested by the Hausman test, the heterogeneity bias is rather insignificant in that sample (see footnote 10).

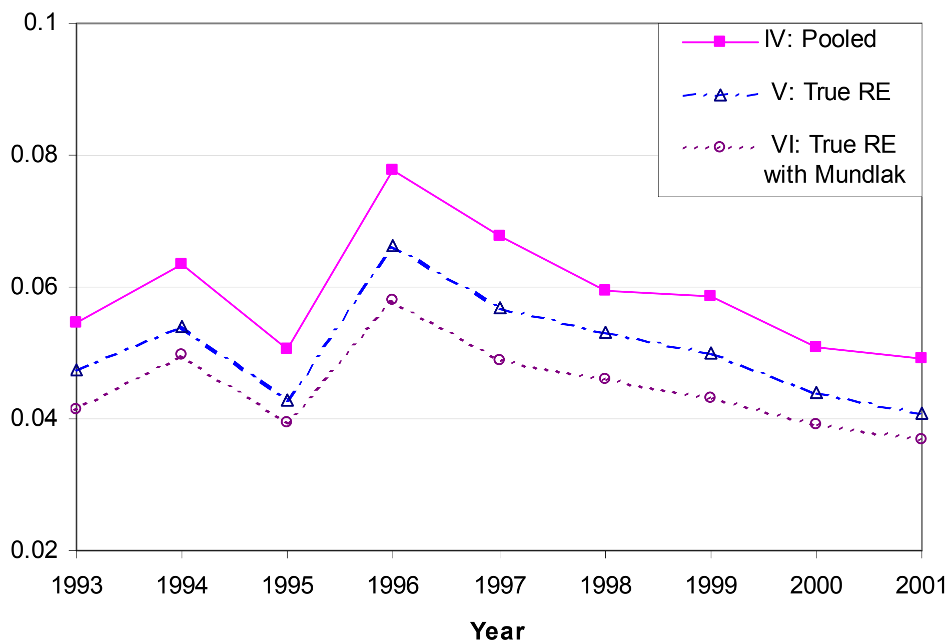
Interestingly, the highest correlation coefficients are observed between models *III* and *VI*, and models *II* and *V*.[33] Both these cases link a time-variant inefficiency model to a model that assumes constant inefficiency. The relatively high correlation between GLS estimates and the true RE model suggests that both models although affected by heterogeneity bias in the coefficients, have a reasonable "mutual consistency" with regard to inefficiency estimation.[34] On the other hand the high correlation between two models with Mundlak's specification suggests that the heterogeneity bias can be resolved without affecting the validity of inefficiency estimates.

33 These models have also the highest correlation coefficients in efficiency ranks (0.98 between *II* and *V*, and 0.95 between *III* and *VI*).

34 The expression "mutual consistency" is used by Bauer et al. (1998) in this context.

Another observation on table 5 is that while the correlation between models *I* and *II* is fairly high (.849), both models show a weak correlation with model *III*, suggesting that Mundlak adjustment in has a significant effect on individual inefficiencies. This pattern is less evident in model *VI* compared to models *IV* and *V*. In fact the Mundlak adjustment does not appear to cause a considerable change in the correlation with the pooled model (*IV*), which is fairly high (about 0.9). However, the correlation between models *V* and *VI* appears to considerably lower when the inefficiencies are averaged over the sample period. This result may suggest that the Mundlak adjustment is not just a shift at the firm level; rather, it causes a differential change in inefficiency estimates of a given firm over time. Similar correlation coefficients have been calculated for efficiency ranks. These coefficients (not shown in the paper) are generally close to the coefficients reported in table 5.

Figure 2
Average inefficiency score



In figure 2 the average inefficiency score is plotted against time, as estimated by models *IV*, *V* and *VI*. All three models suggest that the cost efficiency of Ticino's nursing homes has continuously improved since 1996. As expected, the pooled frontier model slightly overestimates the inefficiencies because it does not consider any firm-specific heterogeneity. This figure shows that the trends estimated by all three models are quite similar. These similarities are the more striking as these models result in significantly different estimates of the cost frontier coefficients (as shown in figure 1). These results, along with similar results in overall average inefficiencies (see table 4), suggest that although these models are different in individual inefficiency scores, the inefficiency estimates have robust average values as long as these values are taken over reasonably large subgroups.[35] This implies that the considerable differences observed in individual scores are induced by sampling variation, rather than by differences in model specification. Therefore, these results points to a general conclusion that the inefficiency estimates in models with time-variant inefficiency are not much sensitive to the correlation between firm-specific heterogeneity and explanatory variables. Such correlations are captured by the cost function coefficients and therefore do not affect the residuals.

7. Concluding remarks

The application of alternative cost frontier models to a panel of nursing homes in Switzerland suggests that the estimated cost frontier is sensitive to the adopted model. In particular, the results largely depend upon how the unobserved heterogeneity among firms is accounted for. Given that in our sample the within-firm variations are significant and that the Hausman test indicates a high risk of heterogeneity bias, the fixed effects (FE) model can be considered as a consistent estimator while the random effects (RE) estimator is likely to be biased. Our analysis indicates that a frontier model with random constant (true RE model) slightly decreases these biases.

The results also point to the weak performance of the FE formulation in estimating inefficiencies in usually small-T panel data samples and in presence of unobserved heterogeneity. Given that in many cases this model is the only unbiased estimator of the cost frontier, a modification that can improve the inefficiency estimates without affecting the model's consistency can prove helpful. In this paper we propose a specification based on Mundlak (1978)'s formulation. This approach allows for a direct control for the potential correlation of firm-specific, latent heterogeneity with explanatory variables. The adjustment has been introduced to the conventional GLS model and the true RE model. The cost function's coefficient estimates have been very close to those of the fixed-effects model, thus unbiased. The advantage over the FE model is that the time-invariant factors as well as other hidden correlations with exogenous variables are disentangled from the inefficiency estimates.

35 Note that each year subgroup has about 36 observations.

Our empirical results suggest that this improvement can be quite significant, especially in models with time-invariant inefficiencies. Overall, the model resulted from combining Mundlak's specification with the true RE model, provides a considerable advantage in that while avoiding heterogeneity bias and improving inefficiency estimates, it allows time-variant inefficiency.

Finally, our individual inefficiency estimates appear rather sensitive to the econometric specification. These differences are partly due to different specifications of inefficiency and heterogeneity across the models and partly to the large sampling errors incurred at the individual level. For instance when inefficiency is time-variant, we have only one observation for each inefficiency estimate; thus large errors can be expected. This problem is documented by Horrace and Schmidt (1996), Street (2003) and Jensen (2003) in both cross-sectional data and small-*T* panels.[36] Obviously, to the extent that inefficiencies remain constant over time a longer panel can help. Nevertheless, the assumption of constant inefficiency can be less realistic in longer panels. Our results indicate that when the inefficiency estimates are averaged over a fairly large number of observations, comparable models give rather similar results, or in case of different outcomes, the differences can be reasonably explained through econometric specification. In particular, the average inefficiency of the sample and the average annual inefficiency rates are consistently similar among three models with time-variant inefficiency.

References

Aigner, D., C. A. K. Lovell and P. Schmidt (1977), 'Formulation and Estimation of Stochastic Frontier Production Function Models', *Journal of Econometrics*, 6, 21-37.

Baltagi, Badi H. (2001) *Econometric Analysis of Panel Data*, 2nd edition, John Wiley and Sons.

Battese, George E. and Tim J. Coelli (1992) "Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India", *Journal of Productivity Analysis*, 3 (1): 153-169.

Battese, George E. and Tim J. Coelli (1988) "Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data", *Journal of Econometrics*, 38: 387-399.

Bauer, Paul W., Allen N. Berger, Gary D. Ferrier, and David B. Humphrey (1998) "Consistency conditions for regulatory analysis of financial institutions: A comparison of frontier efficiency methods", *Journal of Economics and Business*, 50: 85-114.

36 Horrace and Schmidt (1996) show that a panel with 6 periods cannot provide a consistent estimation of individual efficiency scores.

Bös, D. (1986) *Public Enterprise Economics*, Amsterdam, North-Holland.

Chamberlain, Gary (1982) "Multivariate regression models for panel data" *Journal of Econometrics*, 18: 5-46.

Cohen, J. and W.D Spector (1996), "The effect of Medicaid Reimbursement on Quality of Care in Nursing Homes" *Journal of Health Economics*, 15, 23-48.

Cornes, R. (1992), *Duality and Modern Economics,* Cambridge University Press.

Cornwell, Christopher, Peter Schmidt and Robin C. Sickles (1990), "Production frontiers with cross-sectional and time-series variation in efficiency levels" *Journal of Econometrics*, 46 (1990): 185-200.

Farsi, Mehdi and Massimo Filippini (2003) "An empirical analysis of cost efficiency in non-profit and public nursing homes" *Working Paper No. 03-06*, Department of Economics, University of Lugano, Switzerland, February 2003

Filippini, Massimo (2001) "Economies of scale in the Swiss nursing home industry" *Applied Economics Letters*, 8: 43-46.

Friedlaender, A.F. and S.J.Wang Chiang (1983) "Productivity Growth in the Regulated Trucking Industry" *Research in Transportation and Economics*, 1, 149-184.

Greene, William (2004) "Reconsidering Heterogeneity in Panel Data Estimators of the Stochastic Frontier Model", *Journal of Econometrics*, forthcoming.

Greene, William (2003) "Distinguishing between heterogeneity and inefficiency: Stochastic frontier analysis of the World Health Organization's panel data on national health care systems" Working paper, Stern School of Business, New York University (April 2003).

Greene, William (2002a) "Fixed and random effects in stochastic frontier models" Working paper, Stern School of Business, New York University (October 2002).

Greene, William (2002b) "Alternative panel data estimators for stochastic frontier models" Working paper, Stern School of Business, New York University (September 2002).

Hausman, J. A. and W. Taylor (1981) "Panel data and unobservable individual effects" *Econometrica*, 49, 1377-1399.

Hsiao, Cheng (2003) *Analysis of Panel Data*, 2nd edition, Cambridge University Press.

Horarce, W. C., and P. Schmidt (1996) "Confidence statements for efficiency estimates from stochastic frontier models" *Journal of Productivity Analysis*, 7:257-282.

Jensen, Uwe (2000) "Is it efficient to analyse efficiency rankings?" *Empirical Economics*, 25: 189-208.

Jondrow, J., I. Materov, K. Lovell and P. Schmidt (1982) "On the estimation of technical inefficiency in the stochastic frontier production function model" *Journal of Econometrics*, 19, 2/3 (August), 233-238.

Kumbhakar, Subal C. and C. A. Knox Lovell (2000) *Stochastic Frontier Analysis*, Cambridge University Press, Cambridge.

McKay, N.L. (1988), "An Econometric Analysis of Costs and Scale Economies in the Nursing Home Industry" *Journal of Human Resources*, 23, 57-75.

Mundlak, Yair (1978) "On the pooling of time series and cross section data" *Econometrica*, 64 (1): 69-85.

Pitt, M. and L. Lee (1981) "The measurement and sources of technical inefficiency in Indonesian weaving industry" *Journal of Development Economics*, 9: 43-64.

Polachek, S. and B. Yoon (1996) "Panel estimates of a two-tiered earnings frontier" *Journal of Applied Econometrics*, 11, 169-178.

Schmidt, P. and R.E. Sickles, (1984) "Production Frontiers and Panel Data", *Journal of Business and Economic Statistics*, 2, 367-374.

Street, Andrew (2003) "How much confidence should we place in efficiency estimates?" *Health Economics* (forthcoming).

Wagstaff, A. (1989), "Estimating Efficiency in the Hospital Sector: a Comparison of Three Statistical Cost Frontier Models" *Applied Economics*, 21, 659-672.

QUADERNI DELLA FACOLTÀ

I quaderni sono richiedibili (nell'edizione a stampa) alla Biblioteca universitaria di Lugano
via G. Buffi 13 CH 6900 Lugano
e-mail: biblioteca@lu.unisi.ch

The working papers (printed version) may be obtained by contacting the Biblioteca universitaria di Lugano
via G. Buffi 13 CH 6900 Lugano
e-mail: biblioteca@lu.unisi.ch

Quaderno n. 98-01
P. Balestra, *Efficient (and parsimonious) estimation of structural dynamic error component models*

Quaderno n. 99-01
M. Filippini, *Cost and scale efficiency in the nursing home sector : evidence from Switzerland*

Quaderno n. 99-02
L. Bernardi, *I sistemi tributari di oggi : da dove vengono e dove vanno*

Quaderno n. 99-03
L.L. Pasinetti, *Economic theory and technical progress*

Quaderno n. 99-04
G. Barone -Adesi, K. Giannopoulos, L. Vosper, *VaR without correlations for portfolios of derivative securities*

Quaderno n. 99-05
G. Barone-Adesi, Y. Kim, *Incomplete information and the closed-end fund discount*

Quaderno n. 99-06
G. Barone -Adesi, W. Allegretto, E. Dinenis, G. Sorwar, *Valuation of derivatives based on CKLS interest rate models*

Quaderno n. 99-07
M. Filippini, R. Maggi, J. Mägerle , *Skalenerträge und optimale Betriebsgrösse bei den schweizerische Privatbahnen*

Quaderno n. 99-08
E. Ronchetti, F. Trojani, *Robust inference with GMM estimators*

Quaderno n. 99-09
G.P. Torricelli, *I cambiamenti strutturali dello sviluppo urbano e regionale in Svizzera e nel Ticino sulla base dei dati dei censimenti federali delle aziende 1985, 1991 e 1995*

Quaderno n. 00-01

E. Barone, G. Barone-Adesi, R. Masera, *Requisiti patrimoniali, adeguatezza del capitale e gestione del rischio*

Quaderno n. 00-02

G. Barone-Adesi, *Does volatility pay?*

Quaderno n. 00-03

G. Barone-Adesi, Y. Kim, *Incomplete information and the closed-end fund discount*

Quaderno n. 00-04

R. Ineichen, *Dadi, astragali e gli inizi del calcolo delle probabilità*

Quaderno n. 00-05

W. Allegretto, G. Barone-Adesi, E. Dinenis, Y. Lin, G. Sorwar, *A new approach to check the free boundary of single factor interest rate put option*

Quaderno n. 00-06

G.D.Marangoni, *The Leontief Model and Economic Theory*

Quaderno n. 00-07

B. Antonioli, R, Fazioli, M. Filippini, *Il servizio di igiene urbana italiano tra concorrenza e monopolio*

Quaderno n. 00-08

L. Crivelli, M. Filippini, D. Lunati. *Dimensione ottima degli ospedali in uno Stato federale*

Quaderno n. 00-09

L. Buchli, M. Filippini, *Estimating the benefits of low flow alleviation in rivers: the case of the Ticino River*

Quaderno n. 00-10

L. Bernardi, *Fiscalità pubblica centralizzata e federale: aspetti generali e il caso italiano attuale*

Quaderno n. 00-11

M. Alderighi, R. Maggi, *Adoption and use of new information technology*

Quaderno n. 00-12

F. Rossera, *The use of log-linear models in transport economics: the problem of commuters' choice of mode*

Quaderno n. 01-01

M. Filippini, P. Prioni, *The influence of ownership on the cost of bus service provision in Switzerland. An empirical illustration*

Quaderno n. 01-02

B. Antonioli, M. Filippini, *Optimal size in the waste collection sector*

Quaderno n. 01-03

B. Schmitt, *La double charge du service de la dette extérieure*

Quaderno n. 01-04
L. Crivelli, M. Filippini, D. Lunati, *Regulation, ownership and efficiency in the Swiss nursing home industry*

Quaderno n. 01-05
S. Banfi, L. Buchli, M. Filippini, *Il valore ricreativo del fiume Ticino per i pescatori*

Quaderno n. 01-06
L. Crivelli, M. Filippini, D. Lunati, *Effizienz der Pflegeheime in der Schweiz*

Quaderno n. 02-01
B. Antonioli, M. Filippini, *The use of a variable cost function in the regulation of the Italian water industry*

Quaderno n. 02-02
B. Antonioli, S. Banfi, M. Filippini, *La deregolamentazione del mercato elettrico svizzero e implicazioni a breve termine per l'industria idroelettrica*

Quaderno n. 02-03
M. Filippini, J. Wild, M. Kuenzle, *Using stochastic frontier analysis for the access price regulation of electricity networks*

Quaderno n. 02-04
G. Cassese, *On the structure of finitely additive martingales*

Quaderno n. 03-01
M. Filippini, M. Kuenzle, *Analisi dell'efficienza di costo delle compagnie di bus italiane e svizzere*

Quaderno n. 03-02
C. Cambini, M. Filippini, *Competitive tendering and optimal size in the regional bus transportation industry*

Quaderno n. 03-03
L. Crivelli, M. Filippini, *Federalismo e sistema sanitario svizzero*

Quaderno n. 03-04
L. Crivelli, M. Filippini, I. Mosca, *Federalismo e spesa sanitaria regionale : analisi empirica per i Cantoni svizzeri*

Quaderno n. 03-05
M. Farsi, M. Filippini, *Regulation and measuring cost efficiency with panel data models : application to electricity distribution utilities*

Quaderno n. 03-06
M. Farsi, M. Filippini, *An empirical analysis of cost efficiency in non-profit and public nursing homes*

Quaderno n. 03-07
F. Rossera, *La distribuzione dei redditi e la loro imposizione fiscale : analisi dei dati fiscali svizzeri*

Quaderno n. 03-08

L. Crivelli, G. Domenighetti, M. Filippini, *Federalism versus social citizenship : investigating the preference for equity in health care*

Quaderno n. 03-09

M. Farsi, *Changes in hospital quality after conversion in ownership status*

Quaderno n. 03-10

G. Cozzi, O. Tarola, *Mergers, innovations, and inequality*

Quaderno n. 03-11

M. Farsi, M. Filippini, M. Kuenzle , *Unobserved heterogeneity in stochastic cost frontier models : a comparative analysis*