

Fiscal Constitutions¹

Costas Azariadis

University of California, Los Angeles, Los Angeles, California
azariadi@ucla.edu

and

Vincenzo Galasso

IGIER, Universidad Carlos III de Madrid, Madrid, Spain; and CEPR
vincenzo.galasso@uni-bocconi.it

Received October 19, 1998; final version received April 6, 2000;
published online January 30, 2002

We study the impact of fiscal constitutions on intergenerational transfers in an overlapping generation model with linear technology. Transfers represent outcomes of a voting game among selfish agents. Policies are decided one period at a time. Majoritarian systems, which accord voters maximum fiscal discretion, sustain all individually rational allocations, including dynamically inefficient ones. Constitutional rules, which give minorities veto power over fiscal policy changes proposed by the majority, are equivalent to precommitment. These rules eliminate fluctuating and dynamically inefficient transfers and sustain weakly increasing transfer sequences that converge to the golden rule. The golden rule allocation is the unique outcome of Markov constitutional rules. *Journal of Economic Literature* Classification Numbers: D72, H55. © 2002 Elsevier Science (USA)

Key Words: veto power; constitutional rules; indeterminacy.

¹ A partial draft of this paper was circulated under the title “Discretionary Policy and Economic Volatility” in October 1995. We thank Robert Becker, Camelia Bejan, Marco Celentani, V. V. Chari, Russell Cooper, John Driffill, David Levine, Rody Manuelli, Albert Marcet, Ken West, and Bill Zame for helpful comments, without implicating them in the results. Earlier versions of the paper were presented at an NBER Group Meeting on Macroeconomic Complementarities (Stanford University), the Texas Monetary Conference (Texas A&M), the Liverpool–Aberystwyth Workshop on Macroeconomics (University of Liverpool), Tel Aviv University, University of Wisconsin, UCLA, USC, University College–London, the European General Equilibrium Meeting (Barcelona), the Federal Reserve Bank of St. Louis, University of Oregon, University of Washington, the Taiwan National Central University, the Third European Conference on Economic Theory (Santiago de Compostela), and the 14th Latin American Meeting of the Econometric Society (Rio de Janeiro).

1. INTRODUCTION

This paper studies how political constitutions influence intergenerational transfers, in particular, how veto power affects the determinacy and intertemporal efficiency of social security systems.

Indeterminacy is a property of economies in which finitely lived voters are free to change the decisions of their predecessors at zero resource cost. Policymaking looks forward in environments of this type: today's decision depends on expectations of how tomorrow's voters will react to the situation they expect to prevail the day after tomorrow, and so on ad infinitum. Policy choice is indeterminate because there is no way to uniquely pin down the degree of coordination among successive generations of voters.

As a counterpoint to voters' discretion, we propose an environment dominated by a fiscal constitution, which restricts the freedom to alter policies inherited from the past. In particular, a constitution that gives current voters or policymakers some veto power over changes in future policies brings to public choices an element of precommitment that helps pin down fiscal policy. Constitutional restrictions also deliver desirable properties of determinacy and optimality claimed for policy "rules" by Friedman [7, 8] and by Kydland and Prescott [15].

The specific policy question we study is the evolution of pure intergenerational social security transfers among finitely lived households in an infinite economy where individual preferences over fiscal transfers are single peaked, and policy conforms to the wishes of a well-defined "median voter" household.² Analysis of social security naturally sheds light on a number of issues related to other intergenerational resource transfers, e.g., public debt, currency, and the generational distribution of the tax burden. As we shall see in later sections, the reasons why societies maintain a social security system stem in part from a social compact and, hence, apply with equal force to issues such as defaulting on public debt and preserving the purchasing power of currency.

Here we limit ourselves to social security in an overlapping generations model of production in which selfish individuals live two periods and consume one private good. Claims on this good are the only asset in the entire economy. We assume away altruistic preferences and the provision of public goods—two key elements in the political economy of fiscal policy³—in order to bring out more clearly the impact of political institutions on intergenerational transfers.

Political institutions in this paper define the authority of the government, that is, of the voters, to tax away income. We study two constitutional

² Ferejohn [6] analyzes some of the pitfalls in the median voter equilibrium concept.

³ These issues are investigated in Tabellini and Alesina [20] and Tabellini [21, 22].

environments that allow more and less of this power. The more discretionary of the two political systems we study is pure majority voting which permits the larger one of the two homogeneous population groups (“young” and “old”) in our economy to reduce transfer payments to zero. The alternative is a constitutional constraint requiring the majority to obtain the approval of the minority to any changes in social security taxes and benefits.

Constitutional limits to fiscal transfers serve as an endogenous mechanism of partial policy commitment, one that encourages cooperation among successive generations of voters. Fiscal constitutions with veto power reduce the indeterminacy of allocations which occurs under majority voting. In particular, the set of subgame perfect equilibria shrinks to monotone, dynamically efficient transfer sequences converging to the golden rule. Even more impressively, veto power shrinks to a unique point the set of Markov perfect equilibria, and that point turns out to be the golden rule.

Additionally, this result is robust to endogenizing the voting structure in order to permit the electorate to choose the political mechanism, simple majority or constitutional veto power, as well as the transfer level.

Section 2 describes a production economy whose subgame perfect equilibria we analyze in Sections 3 and 4 under the majoritarian and constitutional veto power systems. We discuss the literature relating to social security games in Sections 5.

2. A LINEAR PRODUCTION ECONOMY

To analyze the allocative effects of different fiscal policy under alternative political regimes, we start with a simple economic environment in which the government has a socially useful role. The economy is a standard dynamically inefficient overlapping generations model with a linear technology: it consists of an infinite number of two period lived cohorts. At any point in time only two generations are alive; we call them young and old. Individuals are identical within generations.

Agents in cohort $t = 1, 2, \dots$ evaluate consumption bundles $(c_t^t, c_{t+1}^t) \in \mathcal{R}_{++}^2$ by the utility function

$$u_t = U(c_t^t) + \beta U(c_{t+1}^t), \quad (2.1)$$

where c_t^t represents the consumption at time t of the generation born at time t (the young), c_{t+1}^t is the consumption at time $t+1$ of the same generation (the old), and $\beta > 0$ is the time discount factor. The utility

function is concave, twice differentiable, additively separable, and such that $U'(0) = +\infty$.

Agents supply labor inelastically. They are endowed with a non-negative vector (ω_1, ω_2) of efficiency labor units in youth and in old age. Population grows at a rate $n > 0$. Capital depreciates fully each period, and the production function is linear in labor and capital: $Y_t = F(K_t, L_t) = L_t + RK_t$, with $R > 0$.

The social security system consists of a lump sum tax, τ_t , on the young labor income, ω_1 , and a corresponding lump sum, old age pension, P_t . The system is assumed to be balanced every period, and thus $P_t = (1+n)\tau_t$. We identify with $W_t = \omega_1 - \tau_t + [\omega_2 + (1+n)\tau_{t+1}]/R$ the net present value at time t of the lifetime wealth of an agent born at time t . There is no fiat money or public debt in this economy.

Individuals' budget constraints in youth and old age are

$$\begin{aligned} c_t^t &\leq \omega_1 - \tau_t - z_t \\ c_{t+1}^t &\leq \omega_2 + (1+n)\tau_{t+1} + Rz_t, \end{aligned} \quad (2.2)$$

where z_t represents individual savings.

The economic optimization problem is standard. Young individuals take as given the tax sequence, $\{\tau_t, \tau_{t+1}\}$, and choose savings, z_t , to maximize Eq. (2.1) subject to the budget constraints at Eq. (2.2). Then, the economic equilibrium can be defined as follows

DEFINITION 2.1. For a given sequence of taxes, $\{\tau_t\}_{t=0}^{\infty}$, an economic equilibrium is a sequence of allocations and prices, $\{c_t^t, c_{t+1}^t, K_t, R\}_{t=0}^{\infty}$, such that:

(i) the consumer problem is solved for each generation t ; i.e., agents maximize utility with respect to their savings, z_t , subject to the budget constraints in Eq. (2.2);

(ii) the social security budget constraint is balanced every period, $P_t = (1+n)\tau_t$; and

(iii) goods markets clear at all dates; i.e., $Y_t = c_t^t(1+n) + c_{t+1}^t + K_{t+1} \forall t$.

We assume that an economic equilibrium without social security displays dynamic inefficiency and positive aggregate saving when $R = 1+n : 1+n > R > U'(\omega_1)/\beta U'(\omega_2)$.

The institution of social security in this economy allows the government to transfer resources into the future more efficiently than the accumulation of private capital. The lifetime indirect utility of a young individual born at time t , achieved in an economic equilibrium when the interest rate is R , the

social security taxes are (τ_t, τ_{t+1}) , and the present value of lifecycle income is $\omega_1 - \tau_1 + [\omega_2 + (1+n)\tau_{t+1}]/R$, is

$$v'_t(\tau_t, \tau_{t+1}) = \max\{u_t \mid W_t = \omega_1 - \tau_1 + [\omega_2 + (1+n)\tau_{t+1}]/R\} \quad (2.3)$$

whereas the remaining lifetime indirect utility of an old individual at time t , when the social security taxes are (τ_{t-1}, τ_t) , is

$$v_t'^{-1}(\tau_{t-1}, \tau_t) = U(z_{t-1}(\tau_{t-1}, \tau_t)R + \omega_2 + (1+n)\tau_t), \quad (2.4)$$

where z_{t-1} represents the savings in period $t-1$.

We call *laissez-faire* utility the lifetime indirect utility of a young individual born at time t , when there is no social security:

$$v^{LF} = v'_t(0, 0) = \max\{u_t \mid W_t = \omega_1 + \omega_2/R\}. \quad (2.5)$$

A social planner may easily achieve a higher level of lifecycle utility for all generations $t = 0, 1, \dots$ by adopting the superior social security technology. One example is a stationary reallocation that levies on each person a lump sum tax $\tau \in [0, \tau_g]$, where τ_g is the golden-rule transfer, and distributes the proceeds equally among the older generation. The resulting utility, $v'_t(\tau, \tau)$, is higher than the *laissez-faire* utility v^{LF} , because $1+n > R$. Recall that the transfer choice $\tau_g = \arg \max v'_t(\tau, \tau)$ and provides the young with the golden-rule utility $v_g = v'_t(\tau_g, \tau_g)$.

3. FISCAL POLICY UNDER MAJORITY VOTING

In this section, we analyze the more discretionary of the two constitutional environments: majority voting. Elections take place every period, and all voters, young and old, cast a ballot over the current tax, τ_t . Since every agent has zero mass, no individual voter could affect the outcome of the election. To overcome this problem, we assume sincere voting. There is no commitment technology, and thus current voters cannot compel future voters to pay tax for any $s > t$. We now turn to a formal definition of the majority voting game.

An action at time t for a young player is a lump sum tax, $a_t^y \in T = [0, \omega_1]$, and analogously for an old player $a_t^o \in T = [0, \omega_1]$, where the set T of feasible tax ensures that young age consumption is non-negative. At time t , the public history of the game is given by the sequence of taxes until $t-1$: $h_t = (\tau_0, \tau_1, \dots, \tau_{t-1}) \in H_t = [0, \omega_1]^t$. A time t strategy for a young voter is a mapping from the history into the action space, $\sigma_t^y: h_t \rightarrow [0, \omega_1]$, and analogously for an old voter: $\sigma_t^o: h_t \rightarrow [0, \omega_1]$.

Since we want to analyze a majority voting game, in which the political outcome has to be preferred to any other outcome by a majority of voters, we assume that the outcome function is given by the median of the distribution of actions. Because of sincere voting, and within cohort homogeneity, all agents in a cohort (young or old) cast the same vote. Then, since the young constitute a majority of the voters,⁴ the outcome function coincides with the action of the young, $\tau_t = a_t^y$. For a given sequence of actions profiles, $(a_0^y, a_0^o, \dots, a_t^y, a_t^o, a_{t+1}^y, a_{t+1}^o, \dots)$, and corresponding outcomes, $(\tau_0, \dots, \tau_t, \tau_{t+1}, \dots)$, the payoff function of a young player at time t is given by $v_t^y(\tau_t, \tau_{t+1})$, that is, by his or her indirect utility function, as defined at Eq. (2.3). Analogously, the payoff function of an old player at time t is $v_t^{o-1}(\tau_{t-1}, \tau_t)$, as defined at Eq. (2.4).

3.1. Open-Loop Equilibrium

A convenient benchmark to start with is open-loop strategies that depend purely on calendar time and not at all on history.⁵

DEFINITION 3.1. A strategy profile $\sigma = (\sigma_t^y, \sigma_t^o)_{t=0}^\infty$ is an open-loop equilibrium of the majoritarian voting game played by successive generations of voters if (i) σ does not depend on the history of the game, but only on calendar time; and (ii) σ is a Nash equilibrium of the game.

These strategies are independent of the actions of preceding players, both in and out of equilibrium, and hence provide no incentives for cooperation among generations. Hammond [12] and Sjöblom [19], in fact, recognized that the open loop outcome is zero social security; Loewy [16] also found that the open loop equilibrium of a monetary economy shrinks to zero the purchasing power of currency. Therefore, in open loop equilibria the social security technology is not used, and agents obtain the laissez-faire lifecycle utility, v^{LF} .

3.2. Subgame Perfect Equilibria

If selfish voters are to behave in the apparently cooperative fashion that sustains a social security system, they do so from the vantage point of enlightened self-interest, that is, because each cohort is individually better off with a social security system in place than without one. Incentives to coordinate fiscal policies over cohorts of voters may be thought of as social compacts or norms enforced by a system of rewards and punishments.

⁴ The median voter in postwar U.S. presidential elections in an individual whose age varies between 43 and 46 years. This description corresponds to a member of the young generation in a two-cohort economy and of some intermediate generation in a multi-cohort framework such as Cooley and Soares [4] and Galasso [10].

⁵ Fudenberg and Tirole [9, pp. 130–134] discuss open-loop strategies.

Here is an example of how reinforcement works. Cohorts that transfer to the old the resources specified by the norm expect to receive in their own old age a normal payment; cohorts that defect from the norm in their youth expect to receive a zero transfer in old age. Social norms in this example are enforced by a sequence of trigger strategies that connect the decisions of the voters with the behavior of their predecessors. As we know from Kandori [13] and Salant [18], these strategies make cooperation individually rational when it is unfeasible to commit to a future policy course.

Formally, a majoritarian politico-economic equilibrium is a sequence of allocations and prices, $\{c_t^i, c_{t+1}^i, K_t, R\}_{t=0}^\infty$, and a strategy profile, $\sigma = (\sigma_t^y, \sigma_t^o)_{t=0}^\infty$, such that conditions at Definition 2.1 (economic equilibrium) are satisfied given the sequences of lump sum taxes, $\{\tau_t\}_{t=0}^\infty$ in the outcome of σ , and the strategy profile σ is a subgame perfect equilibrium of the majoritarian voting game, with payoff functions $v_t^i(\tau_t, \tau_{t+1})$ for the young and $v_t^{i-1}(\tau_{t-1}, \tau_t)$ for the old.

Then, simple majoritarian systems turn out to sustain any individually rational allocation as a politico-economic equilibrium by the use of an appropriate trigger strategy profile.

PROPOSITION 3.2 (Majoritarian folk theorem). *For every feasible profile of lifecycle utilities $(v_s^*)_{s=t}^\infty$ bounded below by the laissez faire or open-loop equilibrium utility level v^{LF} , there exists a politico-economic equilibrium of the majoritarian social security game that starts at t and pays off v_s^* for all $s > t$.*

Proof. To see this, consider any strategy profile $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ such that:

$$\sigma_s^{y*} = \begin{cases} \tau_s^* & \text{if } \tau_{s-i} = \tau_{s-i}^* \quad \text{for } i = 1, \dots, s-t \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

Since old voters' actions cannot affect the outcome of the game, $(\tau_s^*)_{s=t}^\infty$ is the sequence of taxes associated with any such profile, and the resulting payoffs are $(v_s^*)_{s=t}^\infty$.

It is easy to see that there are no gains from deviating from the above strategies; that is, no young player will be the first to vote for a tax different from the optimal policy $\tau_s \neq \tau_s^*$. It is also easy to see that it is incentive compatible to punish all defectors.

The young best deviation is to vote for zero transfer, $\tau_s = 0$, and the associated payoff is the laissez faire equilibrium utility level v^{LF} , while the payoff from the strategy σ_s^{y*} is v^* which exceeds v^{LF} by construction. Furthermore, the utility of punishing a defector is still the laissez faire equilibrium level v^{LF} which exceeds the utility from not punishing because $v(\tau^*, 0) \leq v^{LF}$ for $\tau^* \geq 0$. Hence, $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ is a subgame perfect profile.

■

Fiscal policies⁶ sustaining politico-economic equilibria under majority voting are ones that make each cohort prefer intergenerational cooperation to the open-loop outcome. Define the function $\phi: T \rightarrow T$, which maps the tax at time s into the tax at time $s+1$ for every $s = t, \dots, \infty$, from the equation $v_s^s(\tau_s^*, \tau_{s+1}^*) = v^{LF}$, and the function

$$\gamma(\tau_s^*) = \begin{cases} \tau_s^* \frac{R}{1+n} & \text{if } R \geq MRS(\omega_1 - \tau_s^*, \omega_2 + (1+n)\tau_{s+1}^*) \\ \phi(\tau_s^*) & \text{if } R < MRS(\omega_1 - \tau_s^*, \omega_2 + (1+n)\tau_{s+1}^*). \end{cases} \quad (3.2)$$

Also call τ_{\max} the fixed point of $\tau_{s+1}^* = \gamma(\tau_s^*)$ in the interval (τ_g, ω_1) . We can now state the following:

PROPOSITION 3.3. (Characterization theorem). *The set of feasible, individually rational fiscal policies $(\tau_s^*)_{s=t}^{\infty}$ which can be supported by a majoritarian politico-economic equilibrium satisfies (i) $\tau_{s+1}^* \geq \gamma(\tau_s^*)$ and (ii) $\tau_s^* \in [0, \tau_{\max}]$.*

As shown in Fig. 1, the map $\gamma(\tau_s^*)$, which connects today's social security tax with the lowest incentive compatible tax for tomorrow, has two fixed points. These are the zero transfer level, and a higher value, τ_{\max} , above which individually rational transfers explode and youth consumption becomes negative in finite time.

Any feasible social security sequence that satisfies the inequalities (i) and (ii) listed in Proposition 3.3 is a politico-economic equilibrium outcome of the majority voting system. Figures 1 and 2 display all these sequences both directly and also in terms of the consumption allocation that corresponds to each pair of taxes in the sequence. Specifically, the set of equilibria contains a continuum of constant sequences, dynamically inefficient sequences bounded away from the golden rule, and cyclical and chaotic sequences generated by the tentlike map drawn in Fig. 3.

⁶The strategy profile proposed above does not support renegotiation proof equilibria. In fact, if the social security system has been dismantled to punish a deviator, all future generations will prefer to reinstitute the system. A renegotiation proof strategy profile that supports the same set of equilibria requires the punishment of odd-numbered (first, third, etc.) successive defectors, in order to deter unprovoked deviations. Additionally, no median voter has to be the first to defect from the equilibrium policy τ , and the best response to defection has to be immediate punishment by the next median voter.

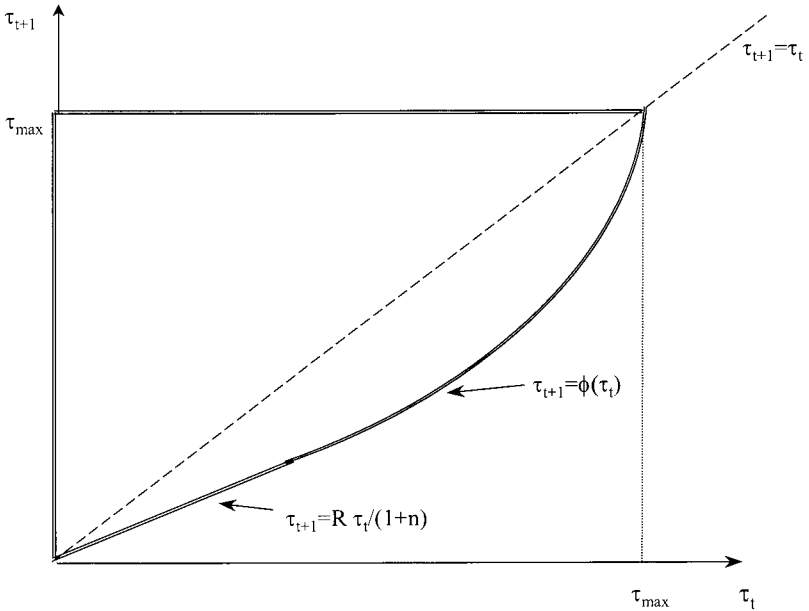


FIG. 1. Set of equilibrium tax rates under majoritarian system.

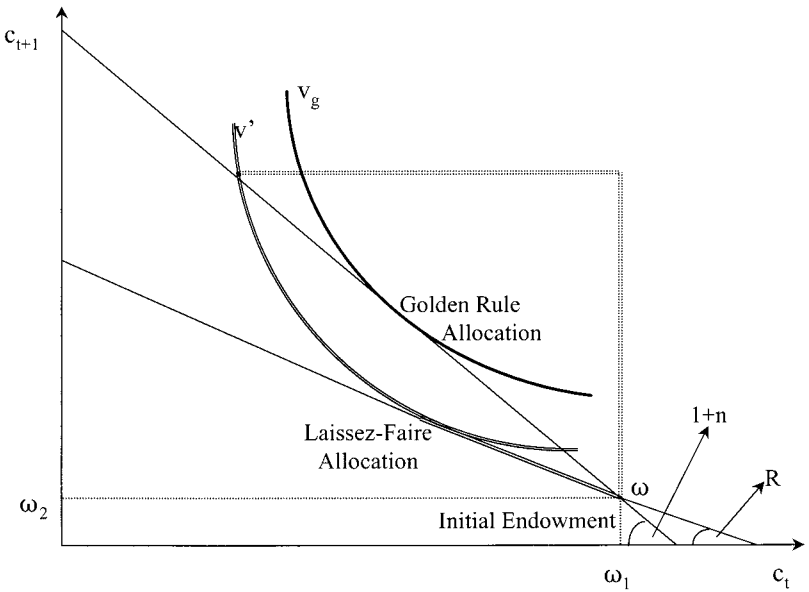


FIG. 2. Set of possible allocations under majoritarian system.

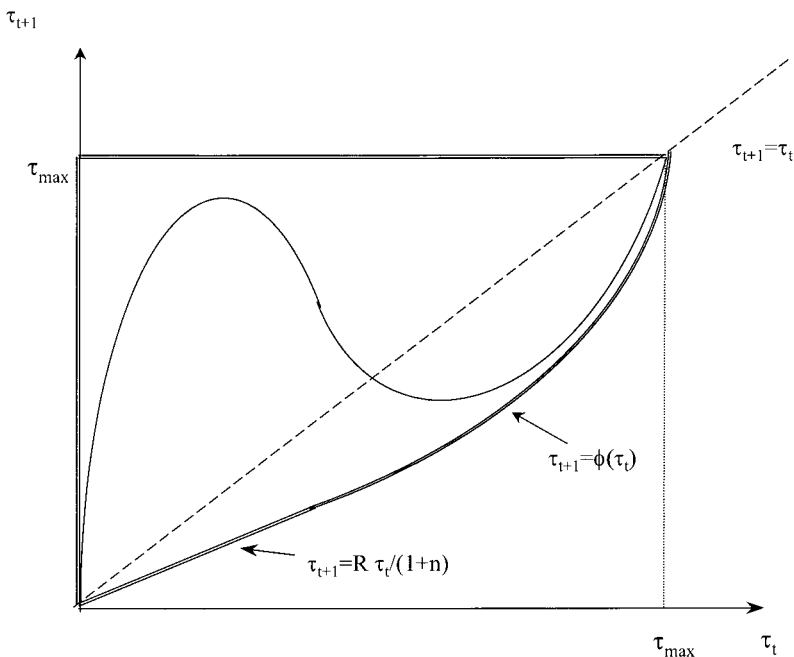


FIG. 3. Cyclical and chaotic tax rate sequences.

The large amount of indeterminacy present in Figs. 1 and 2 stems directly from the inability of voters to commit their successors to a particular course of fiscal policy. Section 4 explores how refinements in political institutions bring about drastic changes in both the size and the volatility of fiscal policies.

4. CONSTITUTIONAL VETO POWER

Large policy adjustments in a democratic society often require wider approval than that of a simple legislative majority. This observation applies particularly when the policy change under consideration contains the seeds of its own reversal because it affects adversely the interests of, and will likely draw loud objection from, a politically significant group.

In what follows we consider a political arrangement that partly pre-commits fiscal policy by awarding the current old voters veto power over policy changes. Veto power is exercised through a constitution, assumed to

be fixed and immutable for the time being. Section 4.2 discusses how this arrangement may come about. In this section, in the spirit of Brennan and Buchanan [3], we will let individuals choose the constitutional constraints under which policymakers have the power to tax them, rather than assuming them, as in the present section.

In every period t , the constitution empowers the younger cohort to propose a tax, $p_t \in T$, which, together with the previous period tax, i.e., the status quo, $\tau_{t-1} \in T$, constitutes the binary fiscal policy agenda, $B_t = \{\tau_{t-1}, p_t\}$. The old are awarded veto power over the young voters' proposal and thus can effectively determine from the agenda the actual policy to be implemented in the current period. As before, all individuals are assumed to vote sincerely.

Like Mulligan and Sala-i-Martin [17], we recognize the success in the political arena of a minority of voters, the elderly. In fact, although in our setting the majority of the voters, the young, retains its control over the fiscal policy; this policy has to be drafted to partially accommodate the preferences of the elderly, since its implementation may draw their objection. Later in this section, we discuss the alternative modeling specification of awarding veto power to the young and empowering the old with agenda setting. The choice of the status quo, rather than some other fiscal policy, e.g., zero transfer, as the policy to which the system reverts if a proposal has been vetoed, is another essential element to establish the political power of the elderly. Notice, for example, that, if the fiscal policy to revert to were zero transfer, the veto power of the old would represent an empty threat, and it would never be used.

We now provide a formal definition of the majoritarian voting game with veto power. An action for a young player at time t is a lump sum tax proposal $a_t^y \in T = [0, \omega_1]$, where the set T of feasible taxes ensures that young age consumption is nonnegative. At time t , an action for an old player is whether or not to veto the young proposal $a_t^o = \{\mathcal{V}, \mathcal{N}\}$. The public history of the game at time t is given by the sequence of taxes until $t-1$: $h_t = (\tau_0, \tau_1, \dots, \tau_{t-1}) \in H_t = [0, \omega_1]^t$. A time t strategy for a young voter is then a mapping from the history into the action space: $\sigma_t^y: h_t \rightarrow [0, \omega_1]$. For an old voter at time t , a strategy is a mapping from the history and from the current young agents' action into the action space: $\sigma_t^o: h_t \times a_t^y \rightarrow \{\mathcal{V}, \mathcal{N}\}$.

In this majoritarian voting game with veto power, the political outcome is the tax preferred by the old between the two taxes on the agenda. Therefore, the outcome function is given by the action of the old, given the action of the young, and the status quo. For a given sequence of actions profiles, $(a_0^y, a_0^o, \dots, a_t^y, a_t^o, a_{t+1}^y, a_{t+1}^o, \dots)$, and corresponding outcomes, $(\tau_0, \dots, \tau_t, \tau_{t+1}, \dots)$, the payoff function for a young player at time t is given by

$v'_i(\tau_i, \tau_{i+1})$, i.e., by his or her indirect utility function, as defined at Eq. (2.3), whereas for an old player at time t the payoff function is $v'^{t-1}(\tau_{t-1}, \tau_t)$, his or her indirect utility as defined at Eq. (2.4).

A politico-economic equilibrium of the majoritarian voting game with veto power is a sequence of allocations, and prices, $\{c'_t, c'_{t+1}, R\}_{t=0}^\infty$, and a strategy profile, $\sigma = (\sigma'_t, \sigma''_t)_{t=0}^\infty$, such that conditions at Definition 2.1 (economic equilibrium) are satisfied given the sequences of lump sum taxes, $\{\tau_t\}_{t=0}^\infty$, in the outcome of σ , and the strategy profile σ is a subgame perfect equilibrium of the majoritarian voting game with veto power, with payoff functions $v'_i(\tau_i, \tau_{i+1})$ for the young and $v'^{t-1}(\tau_{t-1}, \tau_t)$ for the old.

Our constitutional restriction encourages commitment. The old can guarantee themselves the same social security as their immediate predecessors by vetoing any change. The young, too, can ensure a constant fiscal policy sequence by choosing $p_t = \tau_{t-1}$, i.e., with an offer to maintain the status quo.

We assume that the economy starts off in autarky, without a social security system, and the initial agenda in period one is $B_1 = \{0, p_1\}$, $p_1 \geq 0$.

Old generations have a simple decision: they pick the largest item on the agenda because their utility is monotone in the size of the transfer. The old would clearly choose to exercise their power to veto any reduction in social security; hence transfer sequences will be nondecreasing.

Agenda setting by the younger cohort guarantees them the utility associated with a constant tax sequence. In particular, whenever the system is below the golden rule level, $\tau \leq \tau_g$, e.g., starting from autarky at $t = 1$, any young cohort can get a unanimous vote to raise the social security tax level from zero to the golden rule value τ_g and veto any fiscal changes in the subsequent period $t = 2$, thus achieving the golden rule utility v_g . If the system happens to be above the golden rule level at $\tau' > \tau_g$, then any young generation can guarantee itself at most the utility associated with the constant tax τ' , $v(\tau', \tau')$. In the Appendix we show that no young player will propose a tax larger than the golden rule.

Define the family of function indexed by τ , $\psi_\tau: T \rightarrow T$, which maps the tax at time s into the tax at time $s+1$ for every $s = t, \dots, \infty$, from $v'_s(\tau_s^*, \tau_{s+1}^*) = v(\tau, \tau)$, and the function

$$\varphi(\tau_s^*) = \begin{cases} \tau_s^* \frac{R}{1+n} + x & \text{if } R \geq MRS(\omega_1 - \tau_s^*, \omega_2 + (1+n)\tau_{s+1}^*) \\ \psi_{\tau_g}(\tau_s^*) & \text{if } R < MRS(\omega_1 - \tau_s^*, \omega_2 + (1+n)\tau_{s+1}^*), \end{cases} \quad (4.1)$$

where x is such that $\max\{u_s | W_s = \omega_1 + (\omega_2 + x)/R\} = v_g$. It is now straightforward to prove the following analogs of Propositions 3.2 and 3.3. The proofs are in the Appendix.

PROPOSITION 4.1 (Constitutional veto power folk theorem). *For every feasible profile of lifecycle utilities $(v_s^*)_{s=t}^\infty$ whose lower bound is the golden rule level v_g , there exists a politico-economic equilibrium of the majoritarian social security game with veto power that starts at t with zero social security and pays off v_s^* for all $s > t$.*

Fiscal policies that support these payoffs are described in

PROPOSITION 4.2 (Characterization theorem). *The set of feasible, individually rational fiscal policy $(\tau_s^*)_{s=t}^\infty$ which can be supported by a politico-economic equilibrium of the majoritarian game with veto power satisfies (i) $\tau_{s+1}^* \geq \varphi(\tau_s^*)$, (ii) $\tau_{s+1}^* \geq \tau_s^*$, and (iii) $\tau_s^* \in [0, \tau_g]$.*

Once more, part (i) in Proposition 4.2 solves the young voter's individual rationality constraint

$$v_s^s(\tau_s^*, \tau_{s+1}^*) = \max\{u_s \mid W_s = \omega_1 - \tau_s + \omega_2 + (1+n) \tau_{s+1} / R\} \geq v_g \quad (4.2)$$

with the equal sign holding. The function ψ_{τ_g} —interpreted again as a map between today's actual tax and tomorrow's minimum incentive compatible tax—has only one fixed point this time, just τ_g , as Figs. 4 and 5 show.

Since no stationary allocation can pay more than the golden rule, one consequence of the individual rationality constraint in Eq. (4.2) is the following

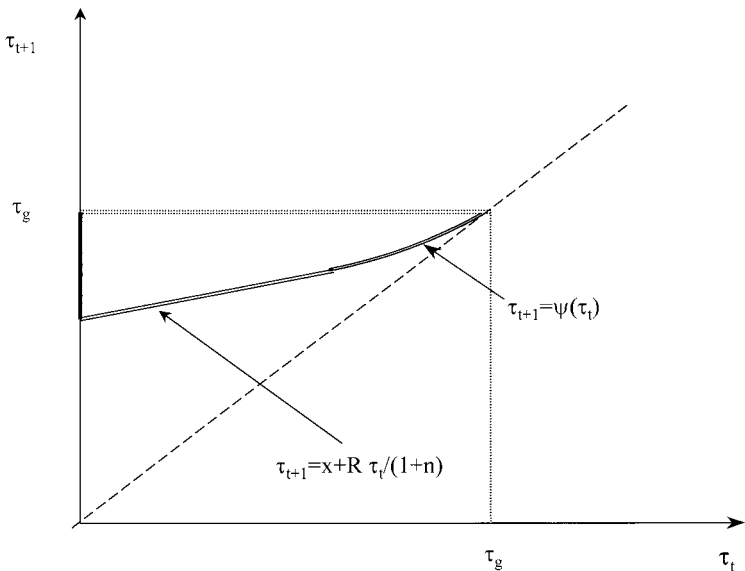


FIG. 4. Set of equilibrium tax rates under constitutional rules.

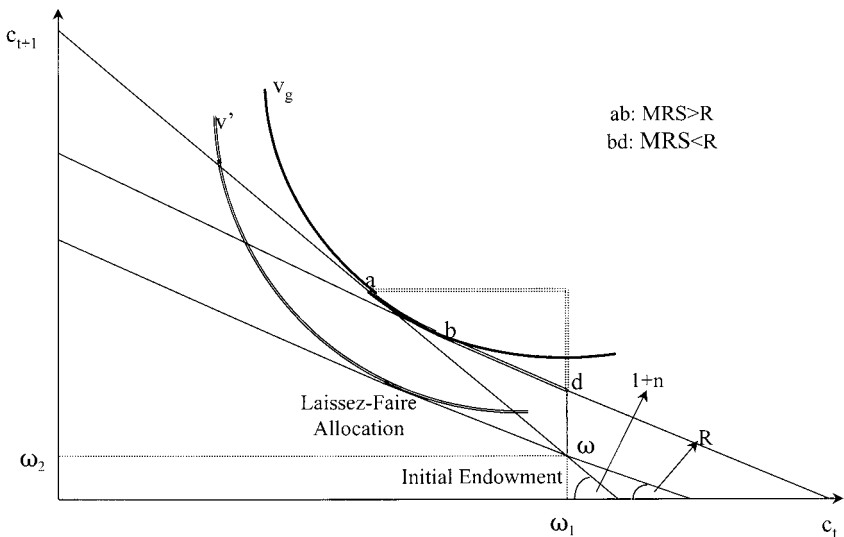


FIG. 5. Set of possible allocations under constitutional rules.

COROLLARY 4.3. *The golden rule is the unique stationary politico-economic equilibrium of the majoritarian game with veto power.*

Of course, it is possible for non-stationary equilibria to pay off more than the golden rule utility because an initial old generation received less than the golden rule transfer. However, the distance of any non-stationary equilibrium from the golden rule must asymptotically shrink to zero. In fact, one easily demonstrates the following result.

PROPOSITION 4.4. *All politico-economic equilibria of the majoritarian game with veto power support allocations that converge to the golden rule.*

An example of a non-stationary equilibrium which gives the initial generation more than the golden rule utility is given by the following transfer sequence,⁷ $\{0, \tau, \tau_g, \dots, \tau_g\}$, such that $v(0, \tau) \geq v_g$, $v(\tau, \tau_g) \geq v_g$, and $v(\tau_g, \tau_g) = v_g$.

The main insight thus is to look at Propositions 4.1 and 4.2 jointly and conclude that a constitutional grant of veto power to the minority is sufficient to eliminate all cyclical sequences and all dynamic inefficiency from majoritarian politico-economic equilibria. Figure 4 shows how much the

⁷ Using logarithmic utility function and initial endowment $(\omega_1, \omega_2) = (1, 0)$, it is easy to show that the above inequalities are satisfied for $[(\frac{1+n}{R})^{\beta/(1+\beta)} - 1] \frac{R}{1+n} \leq \tau \leq \frac{\beta}{1+\beta}$.

policy commitment emanating from this power shrinks the set of equilibrium allocations. Only one steady state survives; all cyclical, chaotic, and dynamically inefficient equilibria disappear.

Let us now consider an alternative specification; the minority, the old generation, set the agenda, and veto power is awarded to the majority, the young. The equilibrium outcomes of the associated voting game would still include weakly increasing sequences converging to the golden rule, as in the previous case, as well as a continuum of constant sequences $\{\tau', \tau', \dots, \tau'\}$ with $\tau' \in [\tau_g, \tau_{\max}]$. Also in this case, all cyclical, chaotic, and dynamically inefficient equilibria disappear; however, there now exist equilibria in which the old can use their agenda setting power to obtain dynamically efficient allocations through transfers which are larger than the golden rule transfer.

4.1. Markovian Fiscal Policies

One way to restrict fiscal policy is to regard it as a stable function of some state variable and not as a sequence which depends on calendar time. These policies restrict economies with different histories but identical structure and capital stock to adopt the same fiscal policy.

Here, we impose this restriction on social security decisions to show how it reduces the indeterminacy of equilibrium allocations by one degree. Moreover, we show that, when coupled with constitutional rules, Markovian policies yield a unique equilibrium allocation, the golden rule.

Young agents solve the economic optimization problem in Section 2, for given values of τ_t and τ_{t+1} , and obtain the optimal individual savings, z_t . The decision over the taxes is taken by the median voter in each young cohort. We restrict the choice of the median voter to stationary, or Markovian, policy functions of the following type,

$$\tau_t = \begin{cases} \theta(k_t) & \text{for } k_t > 0 \\ A & \text{for } k_t = 0, \end{cases} \quad (4.3)$$

where k_t is the stock of capital per capita in the economy, and $\theta(\cdot)$ is a stationary, differentiable function. The accumulation relation equates the net per capita private wealth, Z_t , to the stock of capital: $K_{t+1} = (1+n) Z_t$. Clearly, in equilibrium, net per capita private wealth Z_t has to be consistent with individual savings, z_t .

Thus, the median voter's optimal decision can be obtained by maximizing his or her lifecycle utility with respect to the policy function $\theta(k_t)$, for a given level of the total stock of capital per capita in the economy at time t , k_t , and given expectations on the next period policy function, $\theta^e(k_{t+1})$:

$$\max_{\theta(k_t)} \{u_t \mid W_t = \omega_1 - \theta(k_t) + [\omega_2 + (1+n) \theta^e(k_{t+1})]/R\}. \quad (4.4)$$

We can now define an equilibrium for this Markovian game.

DEFINITION 4.5. A Markovian politico-economic equilibrium is a sequence of allocations, and prices, $\{c'_t, c'_{t+1}, K_t, R\}_{t=0}^\infty$, and a differentiable policy function $\theta(\cdot)$ such that

- (i) conditions at Definition 2.1 (economic equilibrium) are satisfied,
- (ii) net per capita private wealth is equal to individual savings, $K_{t+1} = (1+n)z_t$; and
- (iii) $\theta(\cdot)$ is a fixed point of the mapping from $\theta^e(\cdot)$ to $\theta(\cdot)$, where $\theta(\cdot)$ is the solution to the median voter's problem at Eq. (4.4) and $\theta^e(\cdot)$ is the expected policy function.

We characterize this equilibrium, as well as the dynamics of the economy, for a constant elasticity of substitution utility function: $U(c_t) = c_t^{1-\lambda}/(1-\lambda)$ for $\lambda \neq 1$.

PROPOSITION 4.6 (Characterization theorem). *The set of feasible, individually rational fiscal policy $(\tau_s^*)_{s=t}^\infty$ which can be supported by a Markovian politico economic equilibrium satisfies $\tau_s^* = \theta(k_s) = A - Rk_s$, where A is a free parameter which is pinned down by the first median voter's expectations of future policies.*

The equilibrium policy function can be combined with the saving function implied by a CES utility function and with the accumulation relation to derive the following law of motion for the stock of capital,

$$k_{t+1} = \text{Max} \left\{ 0, \frac{\omega_1 - A}{1+n} - \frac{\omega_2 + A(1+n)}{(1+n)(\beta R)^{1/\lambda}} + \frac{R}{1+n} k_t \right\}$$

$$\text{subject to } \frac{A - \omega_1}{R} \leq k_t \leq \frac{A}{R} \quad \forall t, \quad (4.5)$$

where the restrictions guarantee that $\theta(k_t) \in [0, \omega_1]$.

The capital accumulation dynamics depend on the median voter's expectations through the free parameter A . The capital stock converges to its steady state value

$$k_{ss} = \text{Max} \left\{ 0, \frac{\omega_1 - A}{1+n-R} - \frac{\omega_2 + A(1+n)}{(1+n-R)(\beta R)^{1/\lambda}} \right\}, \quad (4.6)$$

which is positive if $A < A^* < \omega_1$ (see Fig. 6a) and equal to zero if $A \geq A^*$ (see Figs. 6b and 6c).

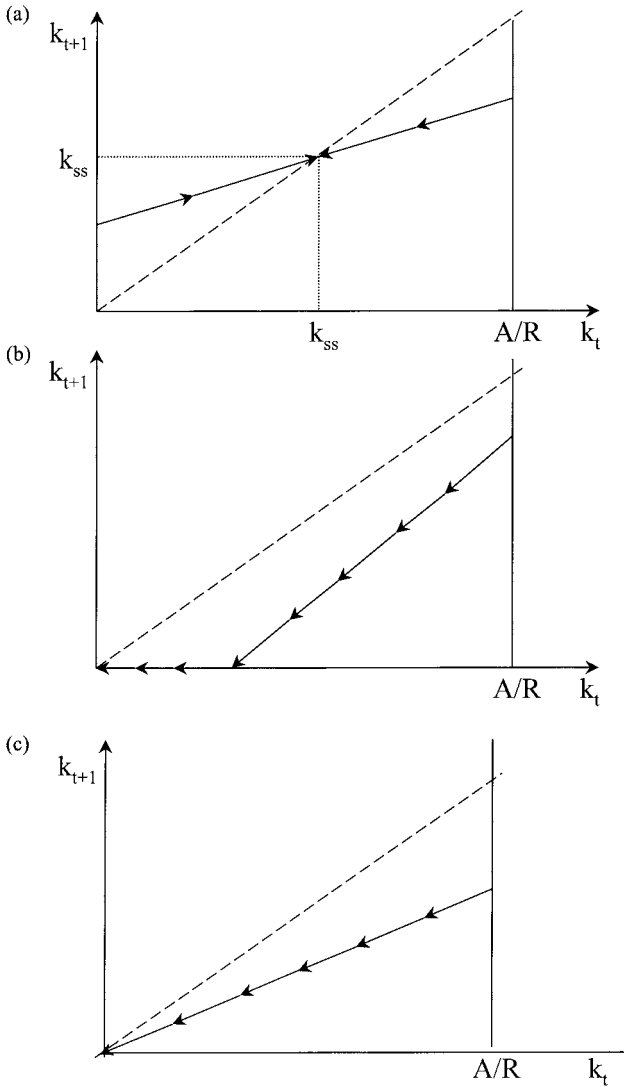


FIG. 6. Capital accumulation. (a) $A < A^*$; (b) $A > A^*$; (c) $A = A^*$.

The evolution of capital has its mirror image in the dynamics of equilibrium taxes. In fact, using the Markovian policy function in Proposition 4.6, and the law of motion for the stock of capital in Eq. (4.5), we can write the law of motion of the equilibrium tax as follows:

$$\theta_{t+1} = \text{Max} \left\{ A, \frac{(\beta R)^{1/\lambda} + R}{(\beta R)^{1/\lambda}} A - \frac{R}{1+n} \frac{(\beta R)^{1/\lambda} \omega_1 - \omega_2}{(\beta R)^{1/\lambda}} + \frac{R}{1+n} \theta_t \right\}. \quad (4.7)$$

Depending on the median voter's expectations through the parameter A , we have three possible cases. Let

$$A' = \frac{R}{1+n} \frac{(\beta R)^{1/\lambda} \omega_1 - \omega_2}{(\beta R)^{1/\lambda} + R}$$

be the value that sets to zero the intercept of the law of motion of the equilibrium tax at Eq. (4.7), and $A^* = ((\beta R)^{1/\lambda} \omega_1 - \omega_2) / ((\beta R)^{1/\lambda} + 1 + n) < \omega_1$ the value that sets to zero the intercept of the law of motion of the capital stock at Eq. (4.5). If $A > A^* > A'$, then the equilibrium social security tax sequence is monotonically increasing to its maximum value, $\theta = A$, as Fig. 7a shows. In this case, to have $\theta(k_t) \in [0, \omega_1]$, a restriction on the parameter A needs to be imposed: $A \leq \omega_1$. If $A > A'$ and $A < A^*$, then the sequence converges to its steady state value

$$\theta_{ss} = \frac{1+n}{1+n-R} \frac{(\beta R)^{1/\lambda} + R}{(\beta R)^{1/\lambda}} A - \frac{R}{1+n-R} \frac{(\beta R)^{1/\lambda} \omega_1 - \omega_2}{(\beta R)^{1/\lambda}} < A, \quad (4.8)$$

as in Fig. 7b. The last case corresponds to $A < A^* < A'$; here the tax monotonically decreases to zero, as shown in Fig. 7c, and resources are transferred into the future exclusively by accumulating physical capital.⁸

To summarize, for a given initial level of capital stock, the set of equilibrium social security taxes associated with a stationary or Markovian policy function contains: (i) a continuum of decreasing sequences (indexed by the parameter A) converging to a steady state with zero social security and positive capital; (ii) a continuum of monotonically increasing or decreasing sequences (again indexed by A) converging to the steady state level θ_{ss} defined in Eq. (4.8) and to a positive capital stock; (iii) a continuum of monotonically increasing sequences converging to a steady state tax level A and a zero capital stock; and (iv) an increasing sequence converging to the golden rule transfer

$$\theta = A = \tau_g = \frac{(\beta R)^{1/\lambda} \omega_1 - \omega_2}{(\beta R)^{1/\lambda} + 1 + n}. \quad (4.9)$$

The adoption of stationary or Markovian policy functions decreases the intrinsic indeterminacy of these intergenerational transfer schemes to a one

⁸ The parallel between capital accumulation and equilibrium tax dynamics is straightforward. Equilibria with a positive level of capital are clearly inefficient as agents do not fully exploit the superior social security technology. Indeed, even zero capital equilibria might be inefficient as long as the social security tax is lower than the golden rule level. In fact, efficient allocations are obtained for zero capital and $\theta = A \geq \tau_g$.

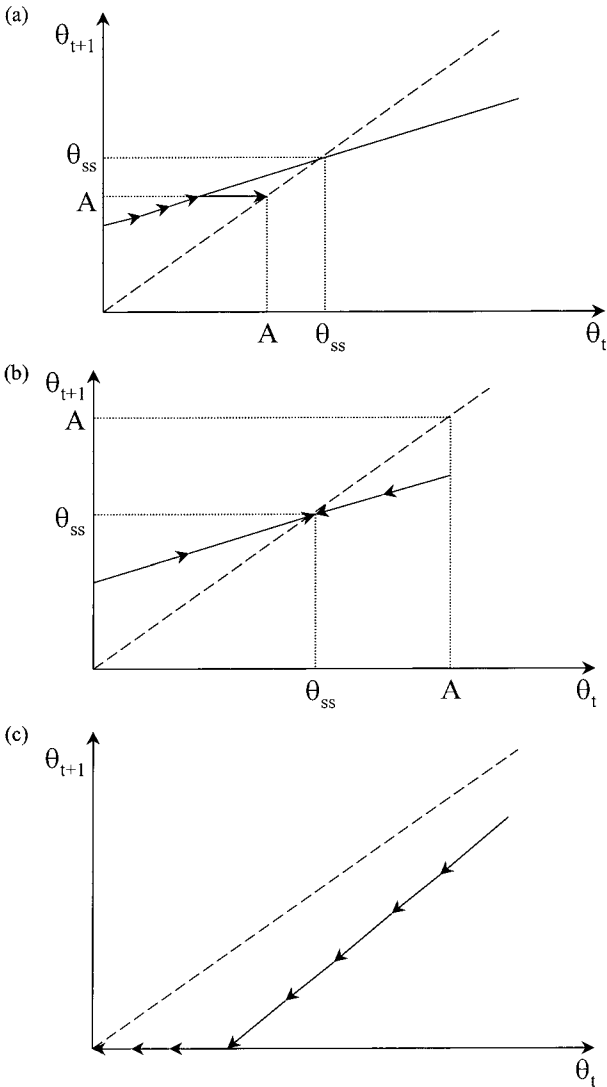


FIG. 7. Markovian equilibrium: tax rate dynamics. (a) $A > A^* > A'$; (b) $A < A^*$, $A > A'$; (c) $A < A' < A^*$.

dimensional indeterminacy. The long run dynamics of the system are indexed on the median voters' expectations through the parameter A , which also determines the efficiency properties of the equilibrium allocations.

One way to allow the median voter to form his or her expectations about future policy is to introduce some degree of commitment, along the line

suggested in the previous section. Specifically, if we restrict the decision space of the voters by introducing veto power,⁹ the free parameter A is pinned down. In this setting, the expectation parameter A equals the extreme right-hand side of Eq. (4.9), and the economy converges to the golden rule with zero capital and a social security tax equal to τ_g .

4.2. *Endogenous Constitutional Choices*

Constitutional awarded veto power represents a commitment device that allows subsequent generations of players to coordinate. But if a society can agree on this constitutional constraint, why can it not agree on efficient outcomes in the first place?

To answer this question, we follow Brennan and Buchanan [3] and endogenize the voting structure. We allow the electorate to choose between a majoritarian and a constitutional veto power system in every period. Fiscal policies may then be thought of as outcomes of a two stage voting game, in which agents decide first the institutional arrangement under which fiscal policy decisions are to be taken and then the level of the transfer. The voting structure chosen in the first stage is adopted both in the second stage election over the tax and in the next period first stage election over the institutional arrangement. Thus, if a society is organized according to constitutional veto power, the consent of the old must be obtained prior to a change in the current transfer level or to a switch, in the next period, from the constitutional veto power to a majoritarian system. A simple majority suffices if there is a majoritarian political system.

Proposition 4.2 generalizes to this setting; that is, weakly increasing transfer sequences converging to the golden rule continue to be equilibria of this two-stage constitutional game. The political system may, however, be indeterminate, because the threat of adopting constitutional restrictions is indeed sufficient to induce the young to partially accommodate the policy to the preferences of the old.

Let us begin with a majoritarian system and zero transfer. Under this system, the initial median voter could obtain a higher utility than the golden rule, either on the transition to the golden rule transfer level (as in some constitutional veto power equilibria) or otherwise. In the former case, the median voter will be indifferent between systems if they both yield the same utility. The latter case is more interesting. Here, the initial median voter could obtain a higher utility than the golden rule on a transfer sequence which will either eventually become larger than the golden rule level, or will eventually decrease. However, neither sequence constitutes an

⁹ In this case the veto power should be interpreted as contingent on the capital level, and therefore it could only be applied to the parameter A . In other words, veto power could be used only if the reduction in the social security tax rate takes place for a given capital stock.

equilibrium of this two stage constitutional game.¹⁰ Alternatively, if the transfer sequence associated with the majoritarian system yields to the initial median voter a lower utility than the golden rule, he or she will prefer the constitutional veto power system. Then, once this constitutional veto power system is in place it will never be changed. In fact, the only transfer sequences that make a median voter strictly better off under the majoritarian system become greater than ω_1 , and thus imply negative youth consumption, in finite time.

To summarize, the potential introduction of a constitutional veto power system acts like a commitment device. In fact, even when it is not put in place, i.e., under a majoritarian system, its mere existence is sufficient to reduce the set of equilibria to non-decreasing sequences converging to the golden rule.

5. RELATED LITERATURE

Social security has a similar role to public debt in reallocating consumption among successive population cohorts. Like public debt and fiat money, social security is a social contrivance whose value as a transfer payment mechanism depends on mutual trust among cohorts and on some degree of intergenerational cooperation. In plain language, social security is like a bubble, and it would be useful to relate the social security equilibria we studied in Sections 3 and 4 with the dynamics of public debt and fiat money we have learned from Wallace [24], Tirole [23], and others.¹¹ The connection is easiest to establish in situations of zero primary budget deficits. Consider, for example, an actuarially fair tax sequence such that

$$-\tau_t + \frac{1+n}{R_{t+1}} \tau_{t+1} = 0, \quad (5.1)$$

where

$$R_{t+1} = \frac{U'(\omega_1 - \tau_t)}{\beta U'(\omega_2 + (1+n) \tau_{t+1})}. \quad (5.2)$$

¹⁰ This can be shown as in Proposition 4.2 some future median voter will not agree to increase the transfer any further (or to decrease it) and will choose to switch to the constitutional system. As usual the argument unravels backward and the initial median voter will choose the constitutional system.

¹¹ See Azariadis [1, Chaps. 19 and 24] for a modern treatment of bubbled dynamics in pure exchange economies.

This sequence adds zero present value to each generation's lifecycle income computed at interest rates that correspond to marginal rates of substitution at the consumption vector $c^t = (\omega_1 - \tau_t, \omega_2 + (1+n)\tau_{t+1})$ implied by the sequence $(\tau_s)_{s=t}^{\infty}$. Each element of this sequence represents excess supply by a typical member of generation t as well as $1/(1+n)$ times the excess demand by each member of generation $t-1$. Equations (5.1) and (5.2), taken together, describe the reflected offer curve of a generation- t household.

Figure 8 reminds us that this curve coincides with the phase diagram of equilibrium in pure-exchange economies with a given stock of fiat money or public debt. All we need to reinterpret actuarially fair social security as public debt or currency is to think of τ_t as the real per capita value of the government liability, and of Eq. (5.1) as the government budget constraint in an economy with zero public consumption and zero primary budget deficit. Then it is easy to see that the golden rule outcome is the only stationary actuarially fair equilibrium and one that is likely to prevail under a credible constitutional arrangement which commits to maintaining the purchasing power of social contrivances—or bubbles—such as currency, public debt, or social security.

By the same token, the indeterminacy of equilibrium we encounter in economies with bubbles is directly related to the absence of a credible promise from the Treasury or the Central Bank to preserve the future value of the bubble. Another source of indeterminacy creeps in if, in addition, we permit governments or median voters to deviate from the "fairness" of the present value relation (5.1) by running a primary budget deficit of their choosing. Then majoritarian equilibria will display the two degrees of indeterminacy exhibited by the subgame perfect allocations of Figs. 1 or 2.

Reducing the large set of subgame perfect equilibria has been a priority in the fiscal policy literature ever since Hammond [12]; it is typically achieved by ruling out trigger strategies. Kotlikoff *et al.* [14] and Esteban and Sakovics [5] restrict the strategy sets of the median voter to costly Markovian strategies of the form $\tau_{t+1} = \phi(\tau_t)$ which assign a fixed resource cost $\alpha > 0$ to any change in the social security tax. The resource cost is a form of partial commitment. For economies starting with zero transfers, the non-cooperative pure-strategy outcome in the case of small α is to reach a transfer somewhat short of the golden rule and to remain there forever.

Grossman and Helpman [11] study a similar problem in a model in which intergenerational transfers are determined by quasi-benevolent politicians, who care about agents' welfare as well as campaign contributions. They show that, also in this setting, there exist multiple Markov perfect equilibria.

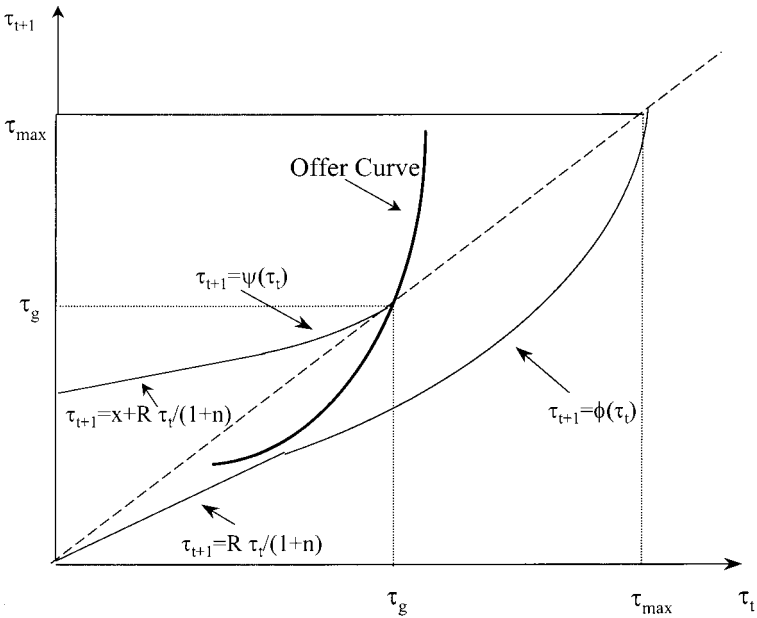


FIG. 8. Equivalence of social security with public debt.

A. TECHNICAL APPENDIX

A.1. Proof of Proposition 3.3

Notice that ϕ is a continuous, increasing, convex function. Then, individual rationality for the strategy profile $(\sigma_s^{y*})_{s=t}^\infty$ implies part (i). Moreover, τ_{s+1}^* is an increasing and quasi-convex function of τ_s^* , with $\phi'(\omega_1) = +\infty$, since $U'(0) = +\infty$. Thus, the function $\gamma(\tau_s^*)$ has another fixed point, $\tau = \tau_{max}$, in the interval (τ_g, ω_1) . Clearly, no equilibrium sequence can go past τ_{max} , or individual rationality would require the sequence to increase until some $\tau > \omega_1$, and youth consumption becomes negative. Finally, notice that the laissez-faire equilibrium represents the threat point that provides the agents with the lowest utility, v^{LF} , and thus the strategy profile $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ at Eq. (3.1) induces the complete set of equilibrium outcomes.

A.2. Proof of Proposition 4.1

Let $A_i^t(\tau) = \{\underbrace{\tau, \tau, \dots, \tau}_i\}$ denote a sequence of constant transfers τ of length i from $t-i$ to t . For a given sequence $\{\tau_0, \tau_1, \dots, \tau_t, \dots\}$, call $i(\hat{A}^t(\tau))$

the length of the longest subsequence of constant transfers τ that finishes at t . Consider the following strategy profile $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ for all the voters, consistent with payoffs $(v_s^*)_{s=t}^\infty$:

$$\sigma_s^{y*} = \begin{cases} \tau_s^* & \text{if } \begin{cases} \text{either } \tau_{s-1} = \tau_{s-1}^* \\ \text{or } \tau_{s-1} = \tau_{s-2} \end{cases} \text{ and } i(\hat{A}^s(\tau_{s-1})) \text{ is even} \\ \tau_{s-1} & \text{otherwise} \end{cases} \quad (\text{A.1})$$

$$\sigma_s^{o*} = \begin{cases} \mathcal{Y} & \text{if } p_s \geq \tau_{s-1} \\ \mathcal{N} & \text{otherwise,} \end{cases} \quad (\text{A.2})$$

where $(\tau_s^*)_{s=t}^\infty$ is the sequence of taxes associated with $(v_s^*)_{s=t}^\infty$. It is straightforward to see that old players have no incentive to deviate from this strategy. For the young, the utility from their best deviation is

$$v_s^D = \max_{\tau \geq \tau_{s-1}^*} v_s^s(\tau, \tau) = \begin{cases} v_g & \text{if } \tau_{s-1}^* \leq \tau_g \\ v_s^s(\tau_{s-1}^*, \tau_{s-1}^*) \leq v_g & \text{if } \tau_{s-1}^* > \tau_g \end{cases} \quad (\text{A.3})$$

and thus $v_s^D \leq v_g$. Moreover, it is incentive compatible to punish a deviator, since $v_s^s(\tau_{s-1}, \tau_{s+1}^*) \geq v_g \geq v_s^s(\tau_s, \tau_s) \forall \tau_s \neq \tau_s^*$ and $\tau_s \geq \tau_{s-1}$, and $v_s^s(\tau_{s-1}, \tau_{s+1}^*) \geq v_s^s(\tau_s, \tau_{s+1}^*)$ for $\tau_s = \tau_s^*$, since $\tau_s^* \geq \tau_{s-1}$. Hence, $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$, which supports v^* , is subgame perfect.

A.3. Proof of Proposition 4.2

Notice that any function ψ_τ indexed by τ is continuous, increasing, and convex. Then, individual rationality for the strategy profile $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ implies respectively (i) and (ii). Moreover, τ_{s+1}^* is an increasing and quasi-convex function of τ_s^* , with $\psi'_\tau(\omega_1) = +\infty \forall \tau$, from $U'(0) = +\infty$. Thus, the function $\varphi(\tau_s^*)$ has only one fixed point, $\tau = \tau_g$, in the interval $(0, \omega_1)$. To show part (iii), consider a sequence of taxes $(\bar{\tau}_s)_{s=t}^\infty$ with $\bar{\tau}_{j-1} < \tau_g < \bar{\tau}_j$. At time j , the young will decide to increase the tax above the golden rule level if $v_j^j(\bar{\tau}_j, \bar{\tau}_{j+1}) > v_g$, which implies $\bar{\tau}_{j+1} \geq \psi_{\tau_g}(\bar{\tau}_j)$. Notice that the young at time $j+1$ can guarantee himself or herself a utility $v_{j+1}^{j+1}(\bar{\tau}_j, \bar{\tau}_j) (< v_g)$. Thus he or she will propose $\bar{\tau}_{j+1} (\geq \psi_{\tau_g}(\bar{\tau}_j))$, if $\bar{\tau}_{j+2} \geq \psi_{\bar{\tau}_j}(\bar{\tau}_{j+1})$, and so on for future players. However, since $\psi'_\tau(\omega_1) = +\infty \forall \bar{\tau} \geq 0$, this is infeasible because some elements of $(\bar{\tau}_s)_{s=t}^\infty$ will become greater than ω_1 in finite time, therefore implying negative consumption. Finally, notice that the golden rule allocation represents the threat point that provides the agents with the lowest utility, v_g , and thus the strategy profile $(\sigma_s^{y*}, \sigma_s^{o*})_{s=t}^\infty$ at Eqs. (A.1) and (A.2) induces the complete set of equilibrium outcomes.

A.4. Proof of Proposition 4.4

The tax sequence $(\tau_s^*)_{s=t}^\infty$ is bounded above by τ_g and weakly increasing. It converges to the golden rule value τ_g because it would be individually irrational for τ_s^* to remain bounded above by a number less than τ_g and unfeasible to remain bounded below by a number bigger than τ_g . The suboptimality is simple to show; if $\tau_s^* < \tau_g$, then Eq. (4.2) tells us that the sequence of taxes has to increase at least at a rate $R/(1+n)$ toward the golden rule. The infeasibility of maintaining τ_s^* some distance above τ_g forever comes again from Part (i) of Proposition 4.2 which requires transfer payments to increase at a rate that approaches $+\infty$ if $\tau_s^* > \tau_g$, since $\psi'_t(\omega_1) = +\infty \forall \tau$.

A.5. Proof of Proposition 4.6

A Markov equilibrium policy function is a fixed point of the mapping from $\theta^e(\cdot)$ to $\theta(\cdot)$, where $\theta(\cdot)$ is the solution to the median voter's problem at Eq. (4.4), and $\theta^e(\cdot)$ is the expected policy function. We assumed the function $\theta(\cdot)$ to be differentiable.

The first order condition for the median voter's maximization problem is

$$-R + (1+n) \frac{\partial \theta(k_{t+1})}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \theta(k_t)} = 0. \quad (\text{A.4})$$

The first term, R , is the marginal cost of increasing today's tax, whereas the second term represents the marginal benefit, which comes from the increase in $\theta(k_t)$ on tomorrow's capital stock. The saving function can be written in terms of lifecycle after-tax endowments $z_t = z[\omega_1 - \theta(k_t), \omega_2 + (1+n)\theta(k_{t+1})]$. Since in an equilibrium at Definition 4.5 we have $z_t = Z_t = (1+n)k_{t+1}$, it follows that

$$(1+n) \frac{\partial k_{t+1}}{\partial \theta(k_t)} = - \frac{z'_1}{1 - z'_2 \frac{\partial \theta(k_{t+1})}{\partial k_{t+1}}}, \quad (\text{A.5})$$

where, for $i = 1, 2$, z'_i represents the derivative of the saving function with respect to the period- i net endowment. Substituting (A.5) into (A.4) and rearranging terms we obtain the differential equation

$$\frac{\partial \theta(k_{t+1})}{\partial k_{t+1}} = - \frac{R}{z'_1 - Rz'_2}. \quad (\text{A.6})$$

Integrating we obtain

$$\theta(k_{t+1}) = A - \frac{R}{z'_1 - Rz'_2} k_{t+1}. \quad (\text{A.7})$$

The constant of integration, A , is a free parameter which is pinned down by the first median voter's expectations of future policies. Any period- t median voter, who expects the next median voter to choose the fiscal policy according to Eq. (A.7), is indifferent between all feasible taxes. This is because a change in θ_t would be exactly compensated by a corresponding change in $\theta_{t+1}^e(k_{t+1})$, driven by a variation in k_{t+1} . Therefore, every median voter is willing to act according to Eq. (A.7), lagged one period, in order to validate the previous median voters' expectations.

REFERENCES

1. C. Azariadis, "Intertemporal Macroeconomics," Blackwell, Oxford, 1993.
2. C. Azariadis and V. Galasso, Discretion, rules, and volatility, *Fed. Res. Bank St. Louis Rev.* **78**, 3 (1996), 65–74.
3. G. Brennan and J. M. Buchanan, "The Power to Tax. Analytical Foundations of a Fiscal Constitution," Cambridge Univ. Press, Cambridge, UK, 1980.
4. T. F. Cooley and J. Soares, A positive theory of social security based on reputation, *J. Polit. Econ.* **107** (1999), 135–160.
5. J. M. Esteban and J. Sakovics, Intertemporal transfer institutions, *J. Econ. Theory* **61** (1993), 189–205.
6. J. Ferejohn, Incumbent performance and electoral control, *Public Choice* **50** (1986), 5–26.
7. M. Friedman, A monetary and fiscal framework for economic stability, *Amer. Econ. Rev.* **38** (1948), 254–264.
8. M. Friedman, The role of monetary policy, *Amer. Econ. Rev.* **58** (1968), 1–17.
9. D. Fudenberg and J. Tirole, "Game Theory," MIT Press, Cambridge, MA, 1991.
10. V. Galasso, The US social security system: What does political sustainability imply?, *Rev. Econ. Dynam.* **3** (1999), 698–730.
11. G. M. Grossman and E. Helpman, Intergenerational redistribution with short-lived governments, *Econ. J.* **108**, 450 (1998), 1299–1329.
12. P. Hammond, Charity: Altruism or cooperative egoism in "Altruism, Morality and Economic Theory" (E. S. Phelps, Ed.), Sage, New York, 1975.
13. M. Kandori, Repeated games played by overlapping generations of players, *Rev. Econ. Stud.* **59** (1992), 81–92.
14. L. J. Kotlikoff, T. Persson, and L. Svensson, Social contracts as assets: A possible solution to the time-consistency problem, *Amer. Econ. Rev.* **78** (1988), 662–677.
15. F. E. Kydland and E. C. Prescott, Rules rather than discretion: The inconsistency of optimal plans, *J. Polit. Econ.* **85** (1977), 473–491.
16. M. Loewy, Equilibrium policy in an overlapping generations economy, *J. Monet. Econ.* **22** (1988), 485–499.
17. C. B. Mulligan and X. Sala-i-Martin, "Gerontocracy," NBER working paper 7117, 1999.

18. D. J. Salant, A repeated game with finitely lived overlapping generations of players, *Games Econ. Behav.* **3** (1991), 244–259.
19. K. Sjöblom, Voting for social security, *Public Choice* **45** (1985), 225–240.
20. G. Tabellini and A. Alesina, Voting on the budget deficit, *Amer. Econ. Rev.* **80** (1990), 37–49.
21. G. Tabellini, “A Positive Theory of Social Security,” NBER Working Paper 3272, 1990.
22. G. Tabellini, The politics of intergenerational redistribution, *J. Polit. Econ.* **99** (1991), 335–357.
23. J. Tirole, Asset bubbles and overlapping generations, *Econometrica* **53** (1985), 1499–1528.
24. N. Wallace, The overlapping generations model fiat money, in “Models of Monetary Economics” (J. Kareken and N. Wallace, Eds.), Federal Reserve Bank of Minneapolis, Minnesota, 1980.