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Positive arithmetic of the welfare state

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Abstract

This paper argues that social security enjoys wider political support than other welfare programs because: (i) retirees constitute the most homogeneous voting group, and (ii) the intragenerational redistribution component of social security induces low-income young to support this system. In a dynamically efficient overlapping generation economy with earnings heterogeneity, we show that, for sufficient income inequality and enough elderly in the population, a welfare system composed of a within-cohort redistribution scheme and an unfunded social security system represents the political equilibrium of a two-dimensional majoritarian election. Social security is sustained by retirees and low-income young; while intragenerational redistribution by low-income young. Unlike unidimensional voting model, our model suggests that to assess how changes in inequality affect the welfare state, the income distribution should be decomposed by age groups.

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1. Introduction

In most industrialized countries, social security represents the single largest item of social welfare expenditure, and a predominant component in the government budget.

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According to OECD data, in 1998, pension transfers represented 61.3% of the total welfare expenditure in Italy, and more than 40% in France, Germany and the UK. Potentially, the welfare state could be composed of several transfer programs targeted to different individual characteristics, such as occupation, location, gender, income and others. Why does the largest social welfare scheme depend on the age of its recipients? Why do we observe so little income redistribution among individuals of the same age group?

This paper argues that social security enjoys wider political support than other welfare programs for two reasons. First, the recipients of social security benefits, the retirees, constitute a homogeneous group, capable of clustering a large block of votes to support this program and to oppose others. Second, the intragenerational redistribution component of the social security system makes this program palatable to low-income young individuals, even when alternative redistribution schemes are available.

The main contribution of this paper is to show that a welfare state composed of a social security system and an income redistribution scheme constitutes the politico-economic equilibrium in a sequence of majoritarian elections when the economy displays sufficiently large labor income inequality, and there are enough elderly in the population. In this equilibrium, the social security system is supported by a voting majority composed of elderly and low-income young, whereas the income redistribution scheme only receives the votes of the low-income young.

It is hardly a new insight to relate the size of the welfare state to the degree of income inequality in the economy. Unidimensional voting models, such as Romer (1975), Roberts (1977) and Meltzer and Richard (1981), suggested that, in democracies, more unequal income distributions induce larger redistribution policies. While we build on this idea, we introduce a further characterization of the agents, their age, to explain the contemporaneous existence of an income-based redistribution scheme and an age-based transfer scheme, the social security system.

Our analysis is motivated by two observations. First, we notice that a large proportion of the earning poor are indeed old individuals. Díaz-Giménez et al. (1997) find that in the US respectively 63% and 28% of the individuals in the first and second earnings quintile are older than 65 years. We argue that these elderly voters may prefer an age-based to an income-based transfer scheme, therefore decreasing the support enjoyed by income redistribution schemes. This is because agents' support to a program may depend on the number of welfare programs they may draw from. In their voting decisions, agents (e.g., retirees) realize that an increase in the size of a (income-based) program will decrease the tax base—via a reduction in the average labor supply—and hence reduce the benefits that they receive from other welfare programs (such as social security). Second, we emphasize the intragenerational redistribution component built in many social security systems.[1] This redistribution from the rich to the poor will play a crucial role in the political equilibrium,

[1] This is due to a combination of contributions, which are typically proportional to the labor income (up to a maximum), and regressive pension benefits. Boskin et al. (1987) and Galasso (2002) provide evidence supporting this view for the US, by showing that, for a given cohort, low-income families obtain larger internal rates of return from investing in social security than middle- or high-income families.

since social security becomes appealing to low-income young, even in the presence of other income redistribution schemes.

We use a dynamically efficient overlapping generation economy with storage technology. Young agents differ in their working ability, and therefore in their labor income. Old individuals do not work. The welfare state consists of two programs that have balanced budgets every period. An (intragenerational) income redistribution scheme taxes labor income and awards a lump sum transfer in young age, whereas an unfunded social security system imposes a payroll tax rate and pays a lump sum pension. The level of the two welfare programs, i.e., the income redistribution and the social security tax rates, is determined in a two-dimensional majority voting game by all agents alive at every election.

Since current voters only decide over current tax rates, in absence of a commitment device over future policies, young voters may have no incentive to support an intergenerational transfer scheme. As most of the literature on political economy of social security, we consider repeated elections, in which implicit contracts among successive generations may arise (see Galasso and Profeta, 2002, for a survey). In these repeated social security games, multiple equilibria typically arise, since several sequences of social security contribution rates may be supported in equilibrium, through the adoption of adequate trigger strategies by the voters. We choose to concentrate on political equilibria induced by institutional restrictions, or structure-induced equilibria, as in Shepsle (1979). By considering issue-by-issue voting over social security and income redistribution tax rates, we are able to restrict the set of equilibrium outcomes to a unique equilibrium, while at the same time dealing with the multidimensionality of the issue space, as suggested by Shepsle (1979). We refer to our equilibrium outcomes as to subgame perfect structure-induced equilibrium (SPSIE) outcomes, as defined in Conde-Ruiz and Galasso (2003). Alternative political arrangements are discussed in Section 5.

We show that, if there is a sufficiently large proportion of elderly in the population and enough income inequality, then a welfare state composed of an income redistribution scheme and an unfunded social security system arises as the structure-induced equilibrium of the majority voting game. In this equilibrium, the social security system is voted by a majority of elderly and low-income young, whereas income redistribution only receives the support of the young voters whose labor income is below the average labor income in the economy.

The idea of a social security system which relies on the political support[2] of low-income young and retirees[3] is due to Tabellini (2000). In his model, low-income, weakly altruistic agents vote for social security since the utility they derive from the pension their parents receive outweighs the direct cost of the social security tax, and an equilibrium with positive social security may arise. However, unlike in our model, this result is not robust to a more complete specification of the welfare state: Were an additional income redistribution scheme to be introduced, the equilibrium would disappear.

[2] See Galasso and Profeta (2002) for a survey of the political economy models of social security.

[3] This aspect has also been addressed by Pestieau (1999), Casamatta et al. (2000) and Persson and Tabellini (2000).

The paper proceeds as follows: Section 2 presents the model and the economic equilibrium. Section 3 develops the political system, and introduces our equilibrium concept. In Section 4, we characterize the equilibria of the voting game. In Sections 5 and 6, we discuss the results and conclude.

2. The model economy

Consider an economy with overlapping generations and a storage technology. Every period two generations are alive, we call them "Young" and "Old". Population grows at a constant rate $\mu>0$. It follows that in any given period t for every young there are $1/(1+\mu)$ old.

Agents work when young, and then retire in their old age. Consumption takes place in old age only. Young individuals differ in their working ability. Working abilities are distributed on the support $[\underline{e}, \bar{e}] \subset \mathcal{R}_+$, according to the cumulative distribution function $G(.)$. An agent born at time t is characterized by a level of working ability and will therefore be denoted by $e_t \in [\underline{e}, \bar{e}]$. The distribution of abilities is assumed to have mean e_ϕ, and to be skewed, $G(e_\phi)>1/2$, which delivers a skewed labor income distribution, as observed in all countries.

A production function transforms labor into the only consumption good, according to the worker's ability:

$$y(e_t) = e_t n(e_t) \tag{2.1}$$

where $n(e_t)$ represents the amount of labor supplied by the agent with ability e_t. A storage technology converts a unit of today's consumption good into $1+R$ units of tomorrow's good, $y_{t+1}=(1+R)y_t$. Since there are no outside assets or fiat money, all private intertemporal transfer of resources takes place through the storage technology. Hence, by assuming that $R>\mu$, we guarantee that the economy is dynamically efficient.

Agents value young age leisure and old age consumption[4] according to a log-linear utility function:

$$U\left(l_t, c_{t+1}^t\right) = \ln(l_t) + \beta c_{t+1}^t \tag{2.2}$$

where l is leisure, c is consumption, β represents the individual discount factor, subscripts indicate the calendar time and superscripts indicate the period when the agent was born.

Young agents face the usual trade off between labor, $n(e_t)$, and leisure, $l(e_t)$, since $n(e_t)=\bar{l}-l(e_t)$, where $\bar{l}$ (>0) is the total amount of disposable time, which we assume to be equal across types. Young pay payroll taxes on their labor income, receive a transfer, and

[4] The assumption that consumption only takes place in old age guarantees the existence of a closed-form solution, but at the cost of abstracting from saving decisions. Cooley and Soares (1999), Galasso (1999) and Boldrin and Rustichini (2000) discuss the relevance of the saving decisions through their impact on the stock of capital, and hence on wages and interest rates, for the political sustainability of social security.

save their disposable income for old age consumption. Old agents have no economic decision to take as they consume their entire income. The lifetime budget constraint for an agent born at time t with ability e_t is then:

$$c_{t+1}^{t} = [e_t n(e_t)(1-\tau_t-\sigma_t)+T_t](1+R)+P_{t+1} \tag{2.3}$$

where τ_t and σ_t are the income redistribution and the social security tax rates at time t, and T_t and P_{t+1} are respectively the young age transfer at time t, and the old age transfer at time t+1.

Young determine their labor supply by maximizing $U(l_t, c_{t+1}^{t})$ with respect to $l(e_t)$ and subject to budget constraint (2.3). We assume that the individual discount factor is equal to the inverse of the interest factor, β=1/(1+R), so that the labor supply does not depend on the interest rate. The optimal labor supply for an ability type e_t agent is then:

$$n(e_t) = \max\left\{0, \bar{l} - \frac{1}{e_t(1-\tau_t-\sigma_t)}\right\}. \tag{2.4}$$

We assume that the labor supply is strictly positive for every type.[5]

Because of the log-linearity of the utility function the labor supply is only affected by changes in the tax rates and not by changes in the transfers level. In this sense, income effects play no role, whereas taxes distort labor supply decisions. This largely simplifies the analysis, because it implies that today's labor supply is not affected by tomorrow's fiscal policies.

2.1. The welfare system

We examine two social welfare instruments, an income redistribution system, and a social security (or pension) system.

The former is an intragenerational redistribution scheme which only affects young generations. In fact, all young persons benefit from a lump sum transfer, T_t , which is financed through a payroll tax, τ_t, on the labor income. Clearly, this system redistributes from rich (above mean income types) to poor (below mean income types) young. A more comprehensive income redistributive scheme would tax all incomes—labor, capital and, possibly, welfare state benefits—and provide the lump sum transfer also to the elderly. We discuss the likely consequences of this more comprehensive program in Section 5. The latter scheme is an intergenerational program, which consists of a sequence of transfers from workers to retirees. Every agent contributes a payroll tax rate, σ_t, from her labor income during her working period, and receives a flat transfer, P_t, when she retires. The combination of these two instruments induces an element of within-cohort redistribution, from the rich to the poor. Finally, every system is assumed to be individually balanced every period, so that its total expenditure has to be equal to the amount of collected taxes.

[5] This assumption amounts to imposing a restriction on the tax rates: $\tau_t+\sigma_t<1-1/\bar{l}\underline{e}$.

The budget constraint at time t for the income redistribution scheme is thus:

$$T_t = \tau_t \int_{\underline{e}}^{\bar{e}} e_t n(e_t) \mathrm{d}G(e_t) \tag{2.5}$$

whereas the budget constraint for the social security system is

$$P_t = \sigma_t (1+\mu) \int_{\underline{e}}^{\bar{e}} e_t n(e_t) \mathrm{d}G(e_t). \tag{2.6}$$

By substituting the labor supply in Eq. (2.4) into Eqs. (2.5) and (2.6), we obtain two new expressions for the welfare system budget constraints:

$$T_t(\tau_t, \sigma_t) = \tau_t \left[e_\phi \bar{l} - \frac{1}{1-\tau_t-\sigma_t} \right] \tag{2.7}$$

and

$$P_t(\tau_t, \sigma_t) = \sigma_t (1+\mu) \left[e_\phi \bar{l} - \frac{1}{1-\tau_t-\sigma_t} \right]. \tag{2.8}$$

The young age lump sum transfer displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the social security payroll tax rate. In fact, the social security tax rate induces a distortion, which contributes to decrease the average income in the economy and thus reduces the young-age benefits. Analogously, the lump sum pension displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the income redistribution tax rate.

2.2. The economic equilibrium

The economic equilibrium can now be defined as follows

Definition 2.1. For a given sequence of tax rates, $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, and a given real interest rate, R, an economic *equilibrium* is a sequence of allocations, $\{l(e_t), c_{t+1}^t(e_t)\}_{e_t \in [\underline{e}, \bar{e}]}^{t=0,\ldots,\infty}$, such that:

- the consumer problem is solved for each generation, i.e., agents maximize $U(l_t, c_{t+1}^t)$ with respect to $l(e_t)$, subject to the restriction in Eq. (2.3);
- the welfare budget constraints are balanced every period, and thus Eqs. (2.5) and (2.6) are satisfied; and
- the aggregate resource constraint is satisfied in every period:

$$\int_{\underline{e}}^{\bar{e}} c_t^{t-1}(e_{t-1}) \mathrm{d}G(e_{t-1}) = (1+R) \int_{\underline{e}}^{\bar{e}} (1-\sigma_{t-1}) e_{t-1} n(e_{t-1}) \mathrm{d}G(e_{t-1}) + \sigma_t (1+\mu) \int_{\underline{e}}^{\bar{e}} e_t n(e_t) \mathrm{d}G(e_t). \tag{2.9}$$

The utility level obtained in an economic equilibrium at time t by an ability type e_t young and by an ability type e_{t-1} old agent can be expressed by their indirect utility functions. For the young:

$$\begin{aligned} v_t^t(\tau_t, \sigma_t, \tau_{t+1}, \sigma_{t+1}, e_t) = & -\ln e_t - 1 - \ln(1 - \tau_t \sigma_t) + e_t \bar{l}(1 - \tau_t - \sigma_t) \\ & + \tau_t \left[e_\phi \bar{l} - \frac{1}{1 - \tau_t - \sigma_t} \right] \\ & + \sigma_{t+1} \frac{1+\mu}{1+R} \left[e_\phi \bar{l} - \frac{1}{1 - \tau_{t+1} - \sigma_{t+1}} \right]. \end{aligned} \tag{2.10}$$

For the old:

$$\begin{aligned} v_t^{t-1}(\tau_{t-1}, \sigma_{t-1}, \tau_t, \sigma_t, e_{t-1}) = & K(\tau_{t-1}, \sigma_{t-1}, e_{t-1}) \\ & + \sigma_t \frac{1+\mu}{1+R} \left[e_\phi \bar{l} - \frac{1}{1 - \tau_t - \sigma_t} \right] \end{aligned} \tag{2.11}$$

where $K(\tau_{t-1}, \sigma_{t-1}, e_{t-1})$ is a constant[6] which depends on the agent's type, but not on current or future tax rates.

These indirect utility functions characterize the young and old agents' preference relations over current (and future) tax rates. Notice that the old individuals' ability type scales their utility up or down, but does not affect their preferences over the tax rates. In other words, all old agents, regardless of their ability type, share the same preferences over welfare programs.

3. The voting game

The amount of welfare expenditures, i.e., income redistribution and social security, is decided in a political process which aggregates the agents' preferences over the two tax rates. We consider majority voting. Elections take place every period, and voters are all agents alive. At every election voters[7] cast their ballots over the two current tax rates, τ_t and σ_t. This majority voting game features two distinguished elements: the issue space is multidimensional and the game is repeated among successive generations of voters.

Since voters only determine current policies, which may be changed at no cost in the future, young individuals may not be willing to support a social security system. In fact, if young agents expect their voting behavior to have no relevance for future choices, they should vote for a zero social security tax rate, or else they would incur in a current labor tax with no future benefits. However, current electors may expect their voting decisions to have an impact on future policies. In this case, as Hammond (1975) initially suggested, an implicit contract among successive generations of voters may arise, in which today's

[6] Specifically, $K(\tau_{t-1}, \sigma_{t-1}, e_{t-1}) = e_{t-1} n(e_{t-1})(1 - \tau_{t-1} - \sigma_{t-1}) + T_{t-1}$.

[7] Since every agent has zero mass, no individual voter would affect the outcome of the election, we hence assume sincere voting.

young agree on a transfer to current retirees because they expect to be rewarded with a corresponding transfer in their old age. A failure to comply with this implicit contract is punished with no old age transfers.

Under these expectations, several sequences of tax rates may indeed be sustained, as these social security games typically generate multiple equilibria. In particular, any sequence of social security and income redistribution tax rates, which—in every period—guarantees to a majority of voters at least the same (lifetime) utility as in the case of no social security, could be supported as a subgame perfect equilibrium outcome. Contributions in the political economy literature of social security have adopted different criteria to select among these equilibria. For instance, Esteban and Sakovics (1993) introduced a transaction cost, Azariadis and Galasso (2002) considered a constitutional veto power, Boldrin and Rustichini (2000) favored the first generation to introduce the system, while in Cooley and Soares (1999) initial voters shared the gains from introducing the system with all future generations of voters. All these games share a common feature, as the issue space is unidimensional.

As in Conde-Ruiz and Galasso (2003), in our multi-dimensional political environment, we decided to concentrate on those equilibrium outcomes, which arise in an issue-by-issue voting game with commitment. This selection device is related to the multidimensionality of the issue space. In a static version of our voting game—obtained by imposing commitment (over social security decisions)—the two-dimensionality of the issue space would typically prevent Nash equilibria from arising.[8] This problem could be dealt with by following Shepsle (1979) in analyzing structure-induced equilibria, in which agents vote simultaneously, yet separately (that is, issue by issue), on the issues at stake. Votes are then aggregated over each issue by the median vote.

This selection criterion has the nice feature of preserving the median voter framework—which we believe to be relevant to analyze these large redistributive programs—even in a multidimensional repeated game, thanks to the notion of issue-by-issue voting. Moreover, to the extent that the structure-induced equilibrium outcome of the voting game with commitment is unique, this selection criterion will single-out a unique equilibrium outcome. We refer to these outcomes as to SPSIE outcomes (a formal definition of the voting game and of the equilibrium is in Appendix A).

To characterize these outcomes, we proceed by analyzing first the structure-induced equilibria of the two-dimensional voting game with commitment over social security. We then relax the assumption of commitment to show that structure-induced equilibrium outcomes of the game with commitment are also subgame perfect equilibrium outcomes of the game without commitment.

In the first stage, we assume full commitment over the tax rate, which finances social security, i.e., the intergenerational redistributive program. Today's voters determine the current and future social security tax rates, however, they only set the current income redistribution tax rate.[9] We consider an issue-by-issue voting game (see Shepsle, 1979), in

[8] See Ordershook (1986) for an extensive review of these issues.

[9] We choose to limit commitment to the social security policy only, because, due to its intergenerational nature, this is the only policy in which commitment can be achieved as an implicit contract among successive generations of voters. We will return to this point in Section 4.3.

which individuals vote contemporaneously, yet separately, over the two issues. The space of alternatives is $(\tau,\sigma)\in[0,1]\times[0,1]$ subject to $\tau+\sigma\leq 1-1/\bar{l}\underline{e}$, which we imposed in order for the labor supply of every young agent to be positive (see Footnote 3). Individual preferences over the two tax rates, as derived from the indirect utility functions at Eqs. (2.10) and (2.11), can easily be shown to be single-peaked if $N=(1+\mu)/(1+R)>1/2$.

In the second stage, we drop the assumption of commitment over future social security policies, and examine individuals' voting strategies, which support the equilibrium outcomes of the voting game under commitment as subgame perfect equilibrium outcomes[10] of the voting game without commitment.

4. Politico-economic equilibria

In this section, we examine the voting game, which determines the size and the composition of the welfare state, (τ, σ). We start by determining the median voters over the two issues, τ and σ, in the voting game with commitment. In particular, we first calculate every elector's ideal point over the current income redistribution tax rate for every given social security tax rates, $\tau(\sigma)$, and then over the current and future social security tax rate for every given income redistribution tax rate, $\sigma(\tau)$. Then, for each value of σ, we identify the median voter over τ, and for each value of τ, the median voter over σ. These median voters' functions intersect at (τ^*, σ^*), which by Proposition 4.1 is a structure-induced equilibrium of the voting game with commitment. We consider the equilibrium of repeated game without commitment in Proposition 4.2.

4.1. Voting on the income redistribution tax rate

A quick look at Eq. (2.11) reveals that the old oppose any income redistribution transfer schemes, since, due to the distortionary taxation, they reduce the average income in the economy—thus decreasing pension benefits—while they fail to provide the old with any transfer.[11] For any positive value[12] of σ, the maximization of Eq. (2.11) with respect to τ hence yields $\tau_{t,\mathrm{old}}=0$.

Young generations, on the other hand, may benefit from this intragenerational transfer scheme, depending on their ability and on the resulting income. For a given social security tax rate, $\sigma_t(=\sigma_{t+1})$, an ability type e_t young at time t would choose her most preferred

[10] Alternatively, we could have examined Markov perfect equilibrium outcomes of the voting game (see Krussell et al., 1997, Grossman and Helpman, 1998, Bassetto, 1999; Hassler et al., 2003), in which the political decisions of the agents are not allowed to depend on the history of the game, and are entirely summarized by the current state variable. In our simple environment with no state variable, we choose to concentrate on stationary subgame perfect equilibria, which deliver a closed form solution (see Azariadis and Galasso, 2002, for a comparison of the Markov and subgame perfect equilibrium outcomes in the context of a social security game).

[11] It can be shown that, even if the income redistribution transfer scheme were to provide a transfer to the old, the elderly voting behavior would not change substantially, provided that the pension transfers were sufficiently large. Here, we abstract from this aspect to simplify the algebra and obtain a closed form solution.

[12] For $\sigma=0$, the old's indirect utility does not depend on τ. Since the old are indifferent, we assume that $\tau^*_{\mathrm{old}}(\sigma=0)=0$.

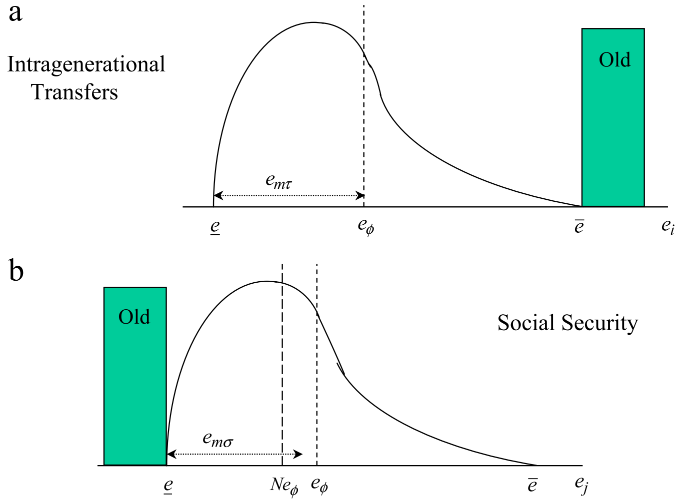

Fig. 1. Voting on intragenerational transfers and social security.

income redistribution tax rate $\tau_{t,e}(\sigma_t)$ by maximizing her indirect utility (see Eq. (2.10)) with respect to τ_t. The first-order condition of this problem yields:

$$\left(e_\phi - e_t\right)\bar{l} - \frac{\tau_t}{\left(1 - \tau_t - \sigma_t\right)^2} = 0.$$

Thus, the optimal income redistribution tax rate, for a given σ, is

$$\tau_{t,e}(\sigma_t) = \max\left\{0, 1 - \sigma_t + \frac{1 - \sqrt{1 + 4\left(e_\phi - e_t\right)\bar{l}(1 - \sigma_t)}}{2\left(e_\phi - e_t\right)\bar{l}}\right\}. \tag{4.1}$$

Unsurprisingly, the young's most preferred income redistribution tax rate is decreasing in their income. Above average income type would vote for τ=0, together with the old. Poor, i.e., below average income, $e_t<e_\phi$, young vote for positive tax rates.

When voting on the income redistribution tax rate, $\tau(\sigma)$, agents can thus be ordered according to their age and income, as shown at Fig. 1a. Since the old generation represents a minority of the total electorate,[13] the median voter on the income redistribution tax rate, hereby intragenerational median voter, is the type $m\tau$ young agent, who divides the electorate in halves, i.e., such that $G(e_{m\tau})=(2+\mu)/2(1+\mu)$. Finally, if the median voter's ability is below the average ability, $e_{m\tau}<e_\phi$, then $\tau_{m\tau}(\sigma)>0$, according to Eq. (4.1), where we drop the time index, since we consider steady states.

[13] Even if we adjust for voting participation rates, retirees are still a minority, although a large and powerful one, see Mulligan and Sala-i-Martin (1999).

4.2. Voting on the social security tax rate

The old have again a simple choice. Since they are no longer required to contribute to the system, they vote for the social security tax rate that maximizes their current transfer (see Eq. (2.11)). For a given income redistribution tax rate, τ, the first-order condition of their optimization problem is

$$e_\phi n(e_\phi) = \frac{\sigma_t}{(1 - \tau_t - \sigma_t)^2} \tag{4.2}$$

where $n(e_\phi)$ represents the average labor supply in the economy, see Eq. (2.4). Their most preferred social security tax rate is thus:

$$\sigma_{t,\text{old}}(\tau_t) = 1 - \tau_t - \sqrt{\frac{1 - \tau_t}{e_\phi \bar{l}}}. \tag{4.3}$$

Because of the assumption of commitment over social security policies, the voting decision of an ability type e_t young individual amounts to maximizing her indirect utility, Eq. (2.10), with respect to the current and future social security tax rate: $\sigma_t = \sigma_{t+1} = \sigma$, and for given values of the current and future income redistribution tax rates, τ_t and τ_{t+1}. The first-order condition yields:

$$e_t n(e_t) - \frac{\partial T_t}{\partial \sigma_t} = \frac{\frac{\partial P_{t+1}}{\partial \sigma_t}}{1 + R} \tag{4.4}$$

where the left-hand side captures the current cost to the young, due to the contribution and to the decrease in the transfer, T_t, driven by a reduction in the tax base, while the right-hand side represents the increase in the future pension benefit. This expression reveals the substitutability between welfare programs: a large income redistribution scheme makes the pension program more costly to the young.

We impose $\tau_t = \tau_{t+1} = \tau$ to restrict our analysis to steady states. Eq. (4.4) can then be rewritten as

$$N e_\phi n(e_\phi) - e_t n(e_t) = \frac{\tau_t + N\sigma_t}{(1 - \tau_t - \sigma_t)^2} \tag{4.5}$$

where $N = (1+\mu)/(1+R)$ can be interpreted as the performance of the social security system relative to the saving (storage) technology. The social security tax rate chosen by a type e young, given an income redistribution tax rate, τ, is then

$$\sigma_{t,e}(\tau_t) = \max\left\{0, 1 - \tau_t + \frac{1 - \sqrt{1 + 4\bar{l}(N + \tau_t(1 - N))(e_\phi N - e_t)}}{2\bar{l}(e_\phi N - e_t)}\right\}. \tag{4.6}$$

This most preferred tax rate, $\sigma_{t,e}(\tau_t)$, is clearly decreasing in the young income type, e_t, because of the within-cohort income redistribution that this scheme achieves through a combination of a proportional income tax, σ_t, and a lump sum pension, P_t. In particular, for sufficiently small values of the income redistribution tax rate, $\tau_t \leq (1-N)/(2-N)$, only those voters whose pre-tax labor income is below a fraction N of the pre-tax average labor income in the economy, $e_t n(e_t) < N e_\phi n(e_\phi)$ (with $N<1$), will vote for a positive social security tax; whereas richer young will oppose the scheme.

A look at Eqs. (4.2) and (4.5) reveals that the old always vote for a larger social security tax than the poorest young, and, hence, than any young. In fact, unlike the young, the old do not make any contribution to the system. Voters' preferences over social security can easily be ordered according to age and income, as displayed at Fig. 1b. The median voter on the social security tax rate is the type $m\sigma$ young who divides the electorate in halves, $G(e_{m\sigma})=\mu/2(1+\mu)$, and thus the median social security tax rate is $\sigma_{m\sigma}(\tau)$ as defined at Eq. (4.6).

4.3. Equilibrium outcomes

In Sections 4.1 and 4.2, we analyzed the voters' decisions over the two welfare schemes, by first determining the decisive or median voter over each issue, $e_{m\tau}$ and $e_{m\sigma}$, and then by calculating their most preferred tax rates, $\tau_{m\tau}(\sigma)$ and $\sigma_{m\sigma}(\tau)$. Eqs. (4.1) and (4.6) may in fact be interpreted as reaction functions. For a given value of the social security (income redistribution) tax rate, Eq. (4.1) (Eq. (4.6)) pins down the income redistribution (social security) tax rate chosen by the median voter $e_{m\tau}$ ($e_{m\sigma}$). Following Shepsle (1979), the (structure-induced) equilibrium outcomes of this voting game correspond to the points where these reaction functions cross.

It is now useful to introduce a measure of the relative ability of the two median voters, $\Delta_\tau=(e_\phi-e_{m\tau})\bar{l}=n(e_\phi)e_\phi-n(e_{m\tau})e_{m\tau}$ and $\Delta_\sigma=(Ne_\phi-e_{m\sigma})\bar{l}$. Notice that, while Δ_τ simply measures the difference between the average labor income in the economy and the intragenerational median voter's labor income, what is relevant in Δ_σ is the difference between the average ability in the economy weighted by the relative performance of the social security system, N, and the social security median voter's ability. This is to take into account that social security is an inferior redistributive scheme for the young, due to its inefficiency in transferring resources into the future. Finally, let Δ be equal to $\Delta_\tau(1-N)-\Delta_\sigma$, and $\hat{\Delta}_\tau$ to $\Delta_\tau-(1-N)\left(1+\sqrt{1+4\Delta_\tau}\right)/2$. The next proposition characterizes the structure-induced equilibrium outcome of the voting game with commitment.[14]

Proposition 4.1. *There exists a unique structure-induced equilibrium of the voting game with commitment over the social security policies, with outcome (τ^*,σ^*), such that*

(I) if $\Delta_\tau\leq 0$ and $\Delta_\sigma \leq -(1-N)$, then $\tau^*=0$ and $\sigma^*=0$;
(II) if $\Delta_\tau\leq 0$ and $\Delta_\sigma > -(1-N)$, then $\tau^*=0$ and $\sigma^*=1+\frac{1-\sqrt{1+4N\Delta_\sigma}}{2\Delta_\sigma}>0$;

[14] It is easy to show that, given an income distribution function, the restrictions on the tax rate imposed earlier, that is, $\tau_t+\sigma_t<1-1/\bar{l}\underline{e}$, are satisfied in equilibrium for sufficiently large values of the total disposable income, $\bar{l}$.

(III) *if* $\Delta_\tau>0$ *and* $\Delta_\sigma \le \hat{\Delta}_\tau$, *then* $\tau^* = 1 + \frac{1-\sqrt{1+4\Delta_\tau}}{2\Delta_\tau} > 0$ and $\sigma^*=0$;
(IV) *if* $\Delta_\tau>0$ *and* $\Delta_\sigma > \hat{\Delta}_\tau$, *then*

$$\tau^* = \Delta_t \frac{1-2N\Delta-\sqrt{1-4N\Delta}}{2\Delta^2} > 0$$

$$\sigma^* = 1 - N - \tau^*\left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right) > 0.$$

A proof is provided in Appendix A. This proposition links the relative ability of the two median voters to the equilibrium welfare state. For sufficiently low levels of income inequality, case I, there are no welfare programs in equilibrium. In case II, the intragenerational median voter's ability is above the mean ability, while the social security median voter's ability is sufficiently low, and thus only the social security system is adopted. This case may arise in an economy with moderate overall income inequality and a large proportion of old voters, or in an economy where the high degree of labor income inequality is mainly due to a large share of retirees. Case III, on the other hand, presents a distribution of income with large inequality in the intragenerational voting, but only small inequality in the social security voting, and thus leads to an equilibrium with income redistribution transfers only. This case may correspond to a young, highly unequal society. Finally, for sufficiently high income inequality, case IV, the equilibrium outcome corresponds to a more comprehensive welfare state, composed of both programs. Fig. 2 illustrates the reaction functions and the equilibrium in case IV, when both systems arise.

The key insight of this proposition is that, to fully appreciate the relation between a welfare system and the labor income inequality in the economy, we need to analyze the underlining income distribution *by age groups*, since age, rather than income, may be the main determinant in some agents' voting decision. In fact, it is straightforward to construct examples in which an increase in income inequality induces a reshuffle in the composition

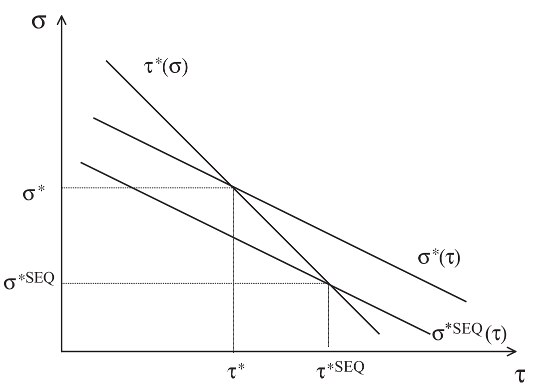

Fig. 2. Equilibrium with intragenerational transfers and social security.

of the welfare state, and may lead to an increase or a reduction in the overall welfare size, depending on whether the population growth rate increases or decreases. Hence, the overall income distribution needs first to be separated in age groups and only then recomposed, as shown in Fig. 1a and b, to take account of income as well as age.

We can now return to the repeated game and drop the assumption of commitment over social security decisions. The next proposition determines the subgame perfect equilibrium outcome of the game resulting from our selection criterion.

Proposition 4.2. *Every pair (τ^*, σ^*), which constitutes a structure-induced equilibrium outcome of the voting game with commitment over the social security policies, is a subgame perfect structure-induced equilibrium outcome of the game with no commitment.*

The idea of the proof, which is provided in the appendix, is simple. Old agents clearly support social security, while poor young support income redistribution. Moreover, low ability young individuals, who would vote for a positive social security level in the game with commitment, will also be willing to enter an implicit contract among successive generations of voters to sustain social security. To illustrate this point, consider the structure-induced equilibrium at case IV in Proposition 4.1, where both tax rates are positive ($\sigma^*>0$, $\tau^*>0$). In this situation, very low ability young individuals would prefer more income redistribution, $\tau'>\tau^*$, and less social security, $\sigma'<\sigma^*$, since the latter scheme is an inefficient redistributive program. However, even if they could affect the voting outcome,[15] they would not be able to change it in the desired direction. In fact, any individual, whose ability is below the median voter's ability, could decrease a tax rate (or both), by voting σ=0 or τ=0 (or σ=τ=0), thus reducing the median tax rate. However, she would not be able to increase the median tax rate, since she is already voting a tax rate larger than the median (voter's) tax rate. A similar reasoning applies to the other voters, and to all cases in Proposition 4.1.

4.3.1. An example of welfare system

To obtain a flavor of the result, we parameterize our simple model to the US economy. Every period corresponds to 25 years. The returns on social security are measured by the product of the real wage growth factor and the population growth factor, which we set respectively equal to 2% and 1.5% annually. The performance of the other saving schemes is indicated by the annual real rate of return, which we set equal to 5.7%, in line with the average real return from the S&P Composite over the last hundred years. It follows that the performance of the social security system relative to other saving schemes is equal to $N=(1+\mu)/(1+R)=(1.015\times1.02)^{25}/(1.059)^{25}=0.6$. This indicates that social security pays out, on average, 40% less than private savings over the lifecycle.

The degree of income inequality is summarized by the relative ability of the two median voters, $e_{m\tau}$ and $e_{m\sigma}$. Using the 1992 Survey of Consumer Finances (SCF) data, we rank individuals[16] according to their ability and age, as shown in Fig. 1a and b, and

[15] Remember that every agent has zero mass.

[16] 1992 Presidential election data are used to account for the different participation rates at elections—and hence for the different political representation—by age and income.

calculate the ratio of the intragenerational and the social security median voter ability to the mean ability. They turn out to be respectively $e_{m\tau}/e_{\phi}$=0.99 and $e_{m\sigma}/e_{\phi}$=0.66. The mean ability in the economy, e_{ϕ}, is normalized to 1, while the total amount of disposable time, $\bar{l}$, is set equal to 5, which, in equilibrium, amounts to an average time spent working of three quarters of the disposable time. The relative ability of the two median voters are thus Δ_{τ}=0.05>0 and Δ_{σ}=−0.3 (>$\hat{\Delta}_{\tau}$=−0.37), additionally Δ=0.32. According to Proposition 4.1, this situation corresponds to case IV, and the associated equilibrium welfare system should thus be composed of positive income redistribution and social security tax rates. In fact, they turn out to be respectively τ*=3.3% and σ*=15.7%. These results are in line with the US welfare system, where the (employee–employer) social security tax rate is 14.4%, and income maintenance programs constitute roughly one sixth of the transfers to the old.

4.4. Equilibrium tax rates and income inequality

In this section, we concentrate on a comprehensive welfare system composed of income redistribution and social security schemes and analyze the effects of changes in income inequality on the equilibrium tax rates. Simple comparative statics show that ceteris paribus a reduction in the ability of the intragenerational median voter shifts up the associated reaction function, $\tau_{m\tau}(\sigma)$, and thus increases the equilibrium income redistribution tax rate while decreasing the social security tax rate. Analogously, an increase in the social security median voter's ability shifts up the other reaction function, $\sigma_{m\sigma}(\tau)$, increases the social security tax rate, and reduces the income redistribution one.

These results, however, are not sufficient to characterize how a change in labor income inequality would affect the equilibrium tax rates for a given age distribution. An increase, for example, in inequality would presumably tend to decrease both median voters abilities with respect to the mean ability in the economy, i.e., Δ_{τ} and Δ_{σ} would increase, and thus would shift both reaction functions in the same direction. The analysis of the consequences on the equilibrium tax rates of such changes represents the object of the next proposition.

First, we decompose the changes in the equilibrium tax rates into the effects due to the variation in the intragenerational median voter's ability (dΔ_{τ}) and in the social security median voter's ability (dΔ_{σ}):

$$\mathrm{d}\tau^* = \underset{(+)}{\frac{\partial \tau^*}{\partial \Delta_{\tau}}}\mathrm{d}\Delta_{\tau} + \underset{(-)}{\frac{\partial \tau^*}{\partial \Delta_{\sigma}}}\mathrm{d}\Delta_{\sigma} \tag{4.7}$$

$$\mathrm{d}\sigma^* = \underset{(-)}{\frac{\partial \sigma^*}{\partial \Delta_{\tau}}}\mathrm{d}\Delta_{\tau} + \underset{(+)}{\frac{\partial \sigma^*}{\partial \Delta_{\sigma}}}\mathrm{d}\Delta_{\sigma}.$$

As previously noted, the direct effect of a change in the median voter's relative ability is positive, whereas the cross effects are negative. The following lemma[17] establishes another useful result.

[17] A proof is in Appendix A.

Lemma 4.3. *For an interior solution of the voting game, (τ*>0, σ*>0),*

$$\left|\frac{\partial \tau^*}{\partial \Delta_\sigma}\right| \le \left|\frac{\partial \sigma^*}{\partial \Delta_\sigma}\right|.$$

The absolute value of the direct effect of a change in the social security median voter's ability is larger than the absolute value of the indirect effect. Finally, let $\eta_{\tau^*,\Delta\sigma} = \frac{\partial \tau^*}{\partial \Delta_\sigma}\frac{\Delta_\sigma}{\tau^*}$ be the elasticity of the equilibrium income redistribution tax rate to changes in Δ_σ, and $\eta_{\sigma^*,\Delta\sigma} = \frac{\partial \sigma^*}{\partial \Delta_\sigma}\frac{\Delta_\sigma}{\sigma^*}$ be the elasticity of the equilibrium social security tax rate to changes in Δ_σ. We can now state the following proposition, which we prove in Appendix A:

Proposition 4.4. *For an interior equilibrium of the voting game, (τ*>0, σ*>0), and for positive changes of Δ_τ and Δ_σ ($d\Delta_\tau$>0, $d\Delta_\sigma$>0), the following holds:*

(i) $d\tau^* \ge 0$ *and* $d\sigma^* \le 0$ *if* $\left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \le \left|\frac{\tau^*/\sigma}{\eta_{\sigma^*,\Delta\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

(ii) $d\tau^* \ge 0$ *and* $d\sigma^* \ge 0$ *if* $\left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right| \le \left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \le \left|-\frac{1}{\eta_{\tau^*,\Delta\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

(iii) $d\tau^* \le 0$ *and* $d\sigma^* \ge 0$ *if* $\left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \ge \left|-\frac{1}{\eta_{\tau^*,\Delta\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

In words, if the increase in income inequality induces a percentage increase in the measure Δ_σ of the social security median voter's ability which is sufficiently smaller than the percentage increase induced in Δ_τ (case i), then the income redistribution tax rate will increase and the social security tax rate will decrease. The opposite happens for percentage increases in Δ_σ sufficiently larger than Δ_τ (case iii). For changes in Δ_σ and Δ_τ of comparable magnitude (case ii), both tax rates increase.

This proposition suggests that, depending on the initial age structure, an increase in income inequality—with no change in the age structure—may indeed lead to a decrease of the overall welfare size, whenever the reduction in the size of one of the program (e.g., the intragenerational transfer) overweights the increase in the other program (e.g., social security).

5. Discussion of the results

The idea that a social security system may rely on the political support of low-income young and retirees was formulated by Tabellini (2000). In his overlapping generation model, heterogeneous (in income), weakly altruistic agents[18] vote every period on the social security level. Young voters do not expect their decision to influence future policy outcomes. Nevertheless, because of their weak altruism, low-income young support social security, since the utility associated to their parents receiving a pension is larger than the direct (utility) cost of the tax. With sufficient income inequality, an equilibrium with social security arises. This equilibrium, however, is not robust to changes in the specification of

[18] Young altruism towards their parents is weak, since they are not willing to give them a direct transfer of resources.

the welfare system. In particular, if a fiscal policy that achieves income redistribution within cohorts is introduced, the equilibrium with social security disappears.

Our paper generalizes this result to a more comprehensive welfare system. In our model, social security may coexist with an income redistribution program in the political equilibrium of a two-dimensional voting game. The intuition is the following. Due to the existence of a within-generation redistributive component in the social security system, low-income young are willing to support both welfare schemes, although they would prefer pure income redistribution to social security. For the retirees, on the other hand, age represents the main determinant in their voting decision. With their votes, they contribute to promote social security and to prevent intragenerational income redistribution schemes from being adopted, since the latter would reduce the tax base in the economy, and hence their pension benefits. Elderly are single-minded (see Mulligan and Sala-i-Martin, 1999), and hence help to shape the two winning majorities. On social security, the majority is composed of retirees and poor young, and the decisive, or median, voter is a low-income young (see Fig. 1b); whereas on income redistribution the decisive, or median, voter is a young agent with a higher labor income than the social security median voter's (see Fig. 1a). The retirees' uniform voting behavior hence contributes to create a wedge between the abilities of the two decisive voters, which is crucial to obtaining an equilibrium welfare system composed of both schemes. The same intuition applies to the analysis of the effects of a change in the overall labor income distribution on the equilibrium tax rates. The final result depends on the impact that the change in income inequality has on the wedge between the two decisive voters, as characterized in Proposition 4.4.

Our results are robust to changes in the specification of the welfare state and of the voting game. Consider a comprehensive income redistribution program, which imposes a proportional tax on all incomes (earnings, capital, transfers and pensions) and pays a lump sum transfer to all agents (young and old). In this case, the elderly would be involved in the income redistribution scheme, because of the contributions on their capital income and pensions, and of the received transfer. However, even if they would become net recipients in this income redistributive program, their voting behavior would be not substantially modified. In fact, to the extent that the pension system is sufficiently large, the reduction induced on the pension transfer by an increase in the income redistribution tax rate would lead the elderly to oppose the income redistributive program.

Would the results change if we adopt a political structure which induces sequential voting? Again, the answer is no. Suppose that elections take place in two rounds. First, agents determine the social security tax rate, σ, and then the income redistribution tax rate, τ. In the first round, voters realize that their decision over σ has a negative effect over τ, because a larger social security system crowds out the level of income redistribution. Since the median voter over σ is a low-income young, she favors a large income redistribution program. Thus, she will vote for a low σ, not to jeopardize the future decision over τ. Graphically, her reaction function $\sigma(\tau)$ is closer to the origin than in the issue-by-issue (simultaneous) voting (see Fig. 2). An analogous of Proposition 4.1 can thus be derived to provide the conditions under which a welfare state composed of both programs represents a politico-economic equilibrium of this sequential voting.

An alternative way of aggregating individual preferences when the issue space is multidimensional is through a model of electoral competition, the probabilistic voting model.[19] In this pre-electoral voting model, candidates commit to an electoral program, which in our setting corresponds to a pair of tax rates (τ, σ). Voters care about the indirect utility associated to these electoral platforms. Additionally, they have idiosyncratic ideological preferences over the candidates. This individual ideology is distributed according to a distribution function, which may vary across group of individuals, such as low or high ability young, and old. On average, individuals are ideologically neutral. Candidates choose their platforms, which in equilibrium turn out to be identical, to maximize the expected probability of being elected. If all groups share the same degree of ideology, i.e., the distributions are identical across groups, the candidates' optimization problem coincides with maximizing a utilitarian utility function. In our setting, the corresponding politico-economic equilibrium would display no income redistribution and positive social security. To obtain an equilibrium welfare state with positive levels of both programs, we would need to assume that the low-income young are more ideologically homogeneous, i.e., their distribution is more concentrate around the mean, than, say, high ability young. In this case, the candidates would optimally place their (identical) programs closer to the ideal point of the low ability young, and would thus choose a positive level of income redistribution.[20]

Finally, if the analysis were carried out in a closed-economy, in which wage and interest rates are endogenously determined, some general equilibrium effects would have emerged to provide additional channels for redistribution. In particular, an increase in the degree of intragenerational redistribution transfers resources from rich to poor agents, and may affect the stock of capital, if individuals differ in their propensity to save. Additionally, as initially suggested by Cooley and Soares (1999), an increase in the size of the social security system tends to reduce private savings, and hence to crowd-out capital, thereby reducing wages, while increasing rates of return. The net effect of these changes on the individual preferences over intragenerational transfers and social security would, however, be uncertain. In fact, while higher returns would boost the elderly's capital income, lower wages would reduce their pension benefits; analogously, young individuals would face lower wages, while enjoying higher returns on their savings.

6. Concluding remarks

Why does the largest US welfare program select its recipients by their age, rather than by their earnings or wealth? In contrast to previous literature, we suggest that a welfare system composed of a PAYG social security program and an income redistribution scheme may represent the political equilibrium of a voting game played by successive generation of voters. In particular, the social security system is supported by a majority of retirees and low-income young.

[19] See Persson and Tabellini (2000) for a survey of all these voting models.

[20] Conversely, if the high ability young are more ideologically homogenous, the equilibrium income redistribution is zero, and social security would be reduced.

Two features are crucial to this result: the political power of the old, which derives from their "extreme" and uniform voting behavior; and the intragenerational redistribution component of the social security system. Unlike young and middle aged individuals, elderly constitute a fairly homogeneous group. They are old, and they have zero (when retired) or low labor earnings, although they may largely differ in their wealth. This homogeneity makes them a uniform electoral block when voting on redistribution issues: they all like social security, and they all may or may not support different forms of income-based redistribution. Since they are able to cluster and shift a large amount of votes, the elderly play a crucial role in shaping the two winning coalitions, as shown in Fig. 1. The existence of a within generation redistribution element in the social security system, on the other hand, induces low-income young to support social security, even in the presence of other income redistribution schemes.

The latter feature bears some implications for the political sustainability of the pension reform strategies recently put forward by several international organizations. Our results suggest that policy measures aimed at reducing the intragenerational component built in the pension system, while at the same time providing more income redistribution, may not be politically viable, because of the opposition of the low-ability young.

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Appendix A

A.1. The voting game

In this section, we provide a formal definition of the voting game and of the equilibrium.

The sequence of social security and income redistribution tax rates until $t-1$ constitutes the public history of the game at time t, $h_t=(\tau_0, \sigma_0), \ldots, (\tau_{t-1}, \sigma_{t-1})\in H_t$, where H_t is the set of all possible history at time t.

An action for a type e young individual at time t is a pair of tax rates, $a^{y}_{t,e}=(\tau, \sigma)\in\Upsilon$, where $\Upsilon = \{(\tau,\sigma) : \tau\in[0,1], \sigma\in[0,1], \tau+\sigma\leq 1 - 1/\bar{l}\underline{e}\}$. Analogously, an action for a type e old individual at time t is $a^{o}_{t,e}=(\tau, \sigma)\in\Upsilon$. We call a_t the action profile of all individuals (young and old) at time t: $a_t=(a^{y}_{t}\cup a^{o}_{t})$ where $a^{y}_{t} = \cup_{e\in[\underline{e},\bar{e}]}\, a^{y}_{t,e}$ and $a^{o}_{t} = \cup_{e\in[\underline{e},\bar{e}]}\, a^{o}_{t,e}$.

For a type e young individual a strategy at time t is a mapping from the history of the game into the action space: $s^{y}_{t,e}: h_t \rightarrow \Upsilon$, and analogously for a type e old individual at time t: $s^{o}_{t,e}: h_t \rightarrow \Upsilon$. The strategy profile played by all individuals at time t is denoted by $s_t = (s^{y}_t \cup s^{o}_t)$ where $s^{y}_t = \cup_{e \in [\underline{e}, \bar{e}]} s^{y}_{t,e}$ and $s^{o}_t = \cup_{e \in [\underline{e}, \bar{e}]} s^{o}_{t,e}$.

At time t, for a given action profile, a_t, the pair (τ^m_t, σ^m_t) represents the medians of the distributions of tax rates. We take (τ^m_t, σ^m_t) to be the outcome function of the voting game at time t. The history of the game is updated according to the outcome function; at time $t+1$: $h_{t+1} = \{(\tau_0, \sigma_0), \ldots, (\tau_{t-1}, \sigma_{t-1}), (\tau^m_t, \sigma^m_t)\} \in H_{t+1}$.

For every agent, the payoff function corresponds to her indirect utility. Formally, for a given sequence of action profiles, $(a_0, \ldots, a_t, a_{t+1}, \ldots)$, and of corresponding realizations, $((\tau_0, \sigma_0), \ldots, (\tau_t, \sigma_t), (\tau_{t+1}, \sigma_{t+1}), \ldots)$, the payoff function for a type e young individual at time t is $v^t_t(\tau_t, \sigma_t, \tau_{t+1}, \sigma_{t+1}, e)$, as defined in Eq. (2.10), and for a type e old agent is $v^{t-1}_t(\tau_t, \sigma_t, e)$, according to Eq. (2.11).

Let $s^{y}_{t|\hat{e}} = s^{y}_t / s^{y}_{t,\hat{e}}$ be the strategy profile at time t for all young individuals except for type $\hat{e}$, and let $s^{o}_{t|\hat{e}} = s^{o}_t / s^{o}_{t,\hat{e}}$ be the strategy profile at time t for all old individuals except for the type $\hat{e}$. Then, at time t, a type $\hat{e}$ young individual maximizes

$$V^t_{t,\hat{e}}\left(s_o, \ldots, \left(s^{y}_{t|\hat{e}}, s^{y}_{t,\hat{e}}\right), s^{o}_t, s_{t+1}, \ldots\right) = v^t_t\left(\tau^m_t, \sigma^m_t, \tau^m_{t+1}, \sigma^m_{t+1}, \hat{e}\right)$$

and a type $\hat{e}$ old individual maximizes

$$V^{t-1}_{t,\hat{e}}\left(s_0, \ldots, \left(s^{o}_{t|\hat{e}}, s^{o}_{t,\hat{e}}\right), s^{y}_t, s_{t+1}, \ldots\right) = v^{t-1}_t\left(\tau^m_t, \sigma^m_t, \hat{e}\right)$$

where, according to our previous definition of the outcome function, (τ^m_t, σ^m_t) and $(\tau^m_{t+1}, \sigma^m_{t+1})$ are, respectively, the medians among the actions over the two welfare programs tax rates played at time t and $t+1$.

We can now define a subgame perfect structure-induced equilibrium of the voting game as follows:

Definition A. 1 *(SPSIE).* A voting strategy profile $s = \{(s^{y}_t \cup s^{o}_t)\}^{\infty}_{t=0}$ is a subgame perfect structure-induced equilibrium (SPSIE) if the following conditions are satisfied: (i) s is a subgame perfect equilibrium; and (ii) at every time t, the equilibrium outcome associated to s is a structure-induced equilibrium of the game with commitment (over social security).

A.2. Proofs of propositions

Proof of Proposition 4.1. Using Eqs. (4.1) and (4.6), it is easy to show that these reaction functions cross only once in the simplex $\tau + \sigma \leq 1$ at (τ^*, σ^*). This is the only point which represents the median among the induced ideal point along both dimensions, τ and σ, and thus, by Shepsle (1979), (τ^*, σ^*) is the only structure-induced equilibrium.

If $\Delta\tau \leq 0$ and $\Delta\sigma \leq -(1-N)$, the reaction functions (4.1) and (4.6) are only defined on the simplex $\tau + \sigma \leq 1$ at $(\tau = 0, \sigma = 0)$. If $\Delta_\tau \leq 0$ and $\Delta_\sigma > -(1-N)$, then $\tau_{m\tau}(\sigma) = 0$, and thus it crosses the reaction function (4.6) on the σ axis at $\sigma^* = 1 + \left(1 - \sqrt{1 + 4N\Delta_\sigma}\right) / 2\Delta_\sigma > 0$.

To find the condition for an interior solution, case iv, notice that for $\Delta_\tau>0$ both reaction functions are negatively sloped, and that $\tau_{m\tau}(\sigma)$ has a higher intercept on the vertical, σ, axis than $\sigma_{m\sigma}(\tau)$. Since both reaction functions are continuous, if $\sigma_{m\sigma}(\tau)$ crosses the horizontal, τ, axis to the right of $\tau_{m\tau}(\sigma)$ there exists a political equilibrium of the voting game for $\tau^*=\Delta_\tau\left(1-2N\Delta-\sqrt{1-4N\Delta}\right)/2\Delta^2$ and $\sigma^*=1-N-\tau^*(2-N-(\Delta_\sigma/\Delta_\tau))$. The condition for the reaction function $\sigma_{m\sigma}(\tau)$ to cross the horizontal axis to the right of $\tau_{m\tau}(\sigma)$ is that $\Delta_\sigma>\hat{\Delta}_\tau=\Delta_\tau-(1-N)\left(1+\sqrt{1+4\Delta_\tau}\right)/2$. If, on the other hand, $\Delta_\sigma<\hat{\Delta}_\tau$, then $\sigma_{m\sigma}(\tau)$ will cross the horizontal, τ, axis to the left of $\tau_{m\tau}(\sigma)$, and thus the equilibrium will be on the horizontal, τ, axis at $\tau^*=1+\left(1-\sqrt{1+4\Delta_\tau}\right)/2\Delta_\tau$. □

Proof of Proposition 4.2. Suppose (τ^*, σ^*) is a structure-induced equilibrium outcome of the voting game with commitment over the social security policies. Let us define the following realization of the public history of the game:

$$H_t^0=\{h_t\in H_t|\sigma_k=0,k=0,\ldots,t-1\}$$

and

$$H_t^\sigma=\{h_t\in H_t|t_0\in\{0,1,\ldots,t-1\}:\sigma_t=0\forall t<t_0 \text{ and } \sigma_t=\sigma^*\forall t\geq t_0\}$$

notice that $H_t^0\cap H_t^\sigma=\emptyset$.

Consider the following strategy $s=(s_{t,e}^{\mathrm{y}}, s_{t,e}^{\mathrm{o}})$, for a type e young:

(i) if $e\leq e_{m\sigma}$

$$s_{t,e}^{\mathrm{y}}=\begin{cases}\left(\tau_{t,e}(\sigma^*),\sigma^*\right) & \text{if} \quad h_t\in H_t^0\cup H_t^\sigma\\ \left(\tau_{t,e}(0),0\right) & \text{if} \quad h_t\in H_t/\left\{H_t^0\cup H_t^\sigma\right\}\end{cases}$$

(ii) if $e>e_{m\sigma}$

$$s_{t,e}^{\mathrm{y}}=\begin{cases}\left(\tau_{t,e}(\sigma^*),\sigma_{t,e}(\tau^*)\right) & \text{if} \quad h_t\in H_t^0\cup H_t^\sigma\\ \left(\tau_{t,e}(0),0\right) & \text{if} \quad h_t\in H_t/\left\{H_t^0\cup H_t^\sigma\right\}\end{cases}$$

and for an old individual

$$s_{t,e}^{\mathrm{o}}=\left(0,\sigma_{t,old}(\tau^*)\right) \text{ if } h_t\in H_t$$

where $\sigma_{t,e_{m\sigma}}(\tau^*)$ is defined in Eq. (4.1); $\tau_{t,e_{m\tau}}(\sigma^*)$ in Eq. (4.6); and $\sigma_{\mathrm{old}}(\tau^*)$ in Eq. (4.3).

Since by definition of SIE, $\sigma^*(\tau^*)=\sigma_{t,e_{m\sigma}}(\tau^*)$, $\tau^*(\sigma^*)=\tau_{t,e_{m\tau}}(\sigma^*)$, it is easy to see that:

$$\begin{aligned}&\tau_{t,e}(\sigma^*)\geq\tau^*\forall e\leq e_{m\tau},\\ &\sigma_{t,e}(\tau^*)\geq\sigma^*\forall e\leq e_{m\sigma}\end{aligned}$$

Recall that the outcome function of the voting game at time t is the median in every dimension of the distribution of actions, (τ_t^m, σ_t^m). No agent has an incentive to deviate from the above strategy. To see this, consider an old individual. She would like to increase the social security tax rate above σ^*, while decreasing τ below τ^*. However, no deviation from the above strategy, $s_{t,e}^{\mathrm{o}}$, may change the equilibrium outcome in this direction, since $\sigma_{t,\mathrm{old}}(\tau^*)$ is already larger than $\sigma^*(\tau^*)=\sigma_{t,e_{m\sigma}}(\tau^*)$, and thus a further increase in $\sigma_{t,\mathrm{old}}(\tau^*)$

would not change the median voter over σ^*, and hence the equilibrium outcome; whereas $\tau_{t,\text{old}}(\sigma^*)$ is equal to zero and thus already below $\tau^*(\sigma^*)=\tau_{t,e_{m\tau}}(\sigma^*)$. It is easy to see that a similar reasoning applies to a type e young individual, with $e<e_{m\sigma}<e_{m\tau}$. With no social security system ever dismissed in the past, $h_t \in H_t^0 \cup H_t^\sigma$, this individual has no incentive to deviate from $\sigma^*(\tau^*)$—and from $\tau^*(\sigma^*)$—since she could not increase the median vote over σ^*—and over τ^*—and hence the equilibrium outcome, as she would like to. As it is common in these games, if the social security system was ever abandoned in the past, no young individual would make a contribution, since future voters will not support the system either. Finally, type e young individuals, with $e>e_{m\sigma}$, will follow their static voting behavior, as described in Eqs. (4.1) and (4.6), if the system was never dropped. These individuals would like to reduce social security spending, however, their vote is already lower than the median vote, $\sigma_{t,e}(\tau^*)<\sigma_t^m(\tau^*)$, and hence no deviation would allow them to decrease $\sigma_t^m(\tau^*)$. The same reasoning applies to the votes over $\tau(\sigma^*)$ both for a type e young—with $e<e_{m\tau}$—who would like to increase $\tau_t^m(\sigma^*)$, and for a young with $e>e_{m\tau}$, who would like to decrease $\tau_t^m(\sigma^*)$. Hence, the previous strategy profile $(s_{t,e}^{\text{y}}, s_{t,e}^{\text{o}})$ constitutes a subgame perfect equilibrium of the voting game with no commitment, with equilibrium outcome (τ^*, σ^*). □

Proof of Lemma 4.3. From Eq. (4.7), $\frac{\partial\sigma^*}{\partial\Delta_\sigma} = \frac{\tau^*}{\Delta_\tau} - \left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right)\frac{\partial\tau^*}{\partial\Delta_\sigma}$ and $\frac{\partial\tau^*}{\partial\Delta_\sigma} = \frac{2}{\Delta}\tau^* + \frac{N\Delta_\tau}{\Delta^2}\left(1 - \frac{1}{\sqrt{1-4N\Delta}}\right)$. Since $(\partial\tau^*/\partial\Delta_\sigma)\leq 0$ and $(\partial\sigma^*/\partial\Delta_\sigma)\geq 0$, thus, it is sufficient to show that $-(\partial\tau^*/\partial\Delta_\sigma)\leq(\partial\sigma^*/\partial\Delta_\sigma)$, which can be done from the previous two expressions and using some simple algebra. □

Proof of Proposition 4.4. To prove part (iii), we use the decomposition at Eq. (4.7) to write $\mathrm{d}\tau^*\leq 0$ and $\mathrm{d}\sigma^*\geq 0$ as $\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau} \geq -\frac{(\partial\tau^*/\partial\Delta_\tau)}{(\partial\tau^*/\partial\Delta_\sigma)}$ and $\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau} \geq -\frac{(\partial\sigma^*/\partial\Delta_\tau)}{(\partial\sigma^*/\partial\Delta_\sigma)}$. From Proposition 4.1, $\frac{\partial\tau^*}{\partial\Delta_\tau} = \frac{\tau^*}{\Delta} - (1-N)\frac{\partial\tau^*}{\partial\Delta_\sigma}\geq 0$ and $\frac{\partial\sigma^*}{\partial\Delta_\tau} = -\frac{\tau^*}{\Delta_\tau} - (1-N)\frac{\partial\sigma^*}{\partial\Delta_\sigma}\leq 0$. Substituting these derivatives in the previous inequality, we obtain: $\left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \geq \left| -\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ and $\left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \geq \left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$. Since by Lemma 4.3, $|1/\eta_{\sigma^*,\Delta_\sigma}|\leq|(\tau^*/\sigma^*)/\eta_{\sigma^*,\Delta_\sigma}|$, then $\mathrm{d}\tau^*\leq 0$ and $\mathrm{d}\sigma^*\geq 0$ for $\left|\frac{\mathrm{d}\Delta_\sigma/\Delta_\sigma}{\mathrm{d}\Delta_\tau/\Delta_\tau}\right| \geq \left| -\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ which prove part (iii). We skip the proof of parts (i) and (ii), which is analogous to part (iii). □

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